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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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Published by The American Society of Mechanical Engineers

October 1950 Revs. 1816–2129 Vol. 3, No. 10

Applied Mechanics Reviews

Published Monthly by The American Society of Mechanical Engineers
at Easton, Pa.

Edited by Midwest Research Institute
with the co-operation of Linda Hall Library
under the sponsorship of

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Editorial Office: APPLIED MECHANICS REVIEWS, Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Mo., USA.

Subscription and Production Office: The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., USA.

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APPLIED MECHANICS REVIEWS, October 1950, Vol. 3, No. 10. Published monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., USA. The editorial office is located at the Midwest Research Institute, Kansas City 2, Mo., USA. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., USA. Cable address "Dynamic," New York. Price $1.50 per copy, $12.50 a year; to members of ASME and co-operating societies $0.75 per copy, $9 a year. Changes of address must be received at Society headquarters three weeks before they are to be effective on the mailing list. Please send old as well as new address..... By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4).... Entered as second-class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1897.....

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

October 1950 Vol. 3, No. 10

The Managing Committee of APPLIED MECHANICS REVIEWS is pleased to announce that, effective September 1, 1950, the editorial offices of the Reviews are located at Midwest Research Institute, 4049 Pennsylvania, Kansas City 2, Missouri. Please address all future correspondence to this address. The Committee extends its appreciation to the Illinois Institute of Technology for excellent and unselfish work during the formative years of the periodical, and for assurance of continued co-operation.

Theoretical and Experimental Methods

(See also Revs. 1845, 1917, 2082)

1816. **Harold Jeffreys and Bertha Swirles Jeffreys, Methods of mathematical physics, second edition,** London and New York: Cambridge University Press, 1950, xii + 708 pp. Cloth, 6.5 × 9 in., $15.

For some research engineers this may be the ultimate mathematical toolbox they have been trying to find. A knowledge of calculus is assumed. The chapter titles are: The real variable, Scalars and vectors, Tensors, Matrices, Multiple integrals, Potential theory, Operational methods, Physical applications of the operational method, Numerical methods, Calculus of variations, Contour integration and Bromwich's integral, Conformal representation, Fourier's theorem, The factorial and related functions, Solution of linear differential equations of the second order, Asymptotic expansions, The equations of potential waves and heat conduction, Waves in one dimension and waves with spherical symmetry, Conduction of heat in one and three dimensions, Bessel functions, Applications of Bessel functions, The confluent hypergeometric function, Legendre and associated functions, Elliptic functions.

The logical temper of the book is defined by the following excerpts from the lively preface: "...careful analysis is more important in science than in pure mathematics, not less... We have also found repeatedly that the easiest way to make a statement plausible is to give a rigorous proof... We have often given proofs under what a pure mathematician will consider unnecessarily restrictive conditions... Generality can be purchased at too high a price. We have given proofs under more general conditions ... when the usual sufficient conditions are not satisfied in practice..."

It is possible to study different chapters concurrently. The book incorporates the familiar *Operational methods* and *Cartesian tensors* of one of the authors. This second edition has been considerably revised. This reviewer has noted with pleasure the following topics: block matrices, an extended treatment of relaxation methods, a translation of Heaviside's method into terms permitting one to decide what is right and what wrong in it, asymptotic expansions group velocity, and others. Some of the topics missed are: stability, nonlinear vibrations, and hyperbolic equations. A. W. Wundheiler, USA

1817. **O. Bottema, A geometrical interpretation of the relaxation method,** Quart. appl. Math. 7, 422–423 (Jan. 1950).

Short note on a geometrical interpretation of one possible set of steps which might be used in the relaxation solution of a system of n linear equations. To obtain a solution to the system $\Sigma_{j=1}^{n} a_{ij}x_j - B_i = 0$, a trial solution Y_i is chosen (arbitrarily), thus defining the residuals $R_i^{(0)}$

$$\Sigma_{j=1}^{n} a_{ij}y_j - B_i = R_i^{(0)}$$

These residuals are changed by altering one x_i at a time. Thus there is set up a sequence of points P_k in Euclidean n-space defined by the R_i. Since the desired solution lies at the origin, it is noted that the change d, may be chosen to make the distance (in R_i-space) a minimum. The d so chosen ($d = -\Sigma a_{ik}R_i/\Sigma a_{ik}^2$) defines the projection of each point P_k on the $(n-1)$ space obtained by the n-space by omitting the altered coordinate.

The affine correspondence thus obtained assures convergence of method if a fixed cyclic order of changes is made. If all $(n-1)$ spaces are mutually perpendicular, the process ends after n steps. Howard W. Emmons, USA

1818. **H. C. Hamaker, The theory of sampling inspection plans,** Philips tech. Rev. 11, 260–270 (Mar. 1950).

Author shows how one goes about actual construction of sampling plans fulfilling certain prescribed conditions. He points out that two fundamental parameters, called "point of control" and "relative slope," really describe the main features of an operating characteristic curve. He shows further (by example) that single and double sampling plans possessing same point of control have almost identical operating characteristics (it would be interesting to prove this result in general). Author shows further how his parameters are related to parameters commonly used by Dodge and Romig, such as producers' and consumers' risks, AOQL, etc. Brief discussion is given of sequential plans. Reviewer believes this article should prove useful to those faced with problem of constructing sampling plans quickly.

Benjamin Epstein, USA

1819. **Donald P. Eckman, Industrial instrumentation,** New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1950, vi + 396 pp. Cloth, 8.50 × 5.75 in., $5.

The book is intended primarily for the undergraduate in engineering. Ed.

1820. Adalbert Duschek and August Hochrainer, Elements of tensor calculus in analytic presentation (Grundzuge der Tensorrechnung in analytischer Darstellung) Part 2: Tensor analysis (in German), Vienna, Springer Verlag, 1950, viii + 338 pp., 64 figs. Paper, 5.5 × 8.25 in., 86.

For part 1 see REV 2, **960**. About 50 pages deal with tensor analysis proper. Remainder consists of differential geometry and potential theory, most of it in tensor notation. Part 3 will contain applications to hydrodynamics, acoustics, and electromagnetism. A. W. Wundheiler, USA

1821. R. Weber, Methods for the computation of eigenvalues and eigenvectors of matrices (in French), Rech. aéronaut. 1949, no. 10, 57–60 (1949).

The author gives a simple elimination procedure, of universal application, for the accurate solution of the title problem. The matrix $A - tB$ is reduced to a triangular form. The reviewer believes that the author is overrating the method, since it requires more operations than that of Danelevskiĭ and Samuelson, and almost as many as a host of other ones. Nevertheless, it represents an essential contribution.

Another method of the paper consists in a previous determination of the eigenvectors x from a linear system of $(n^2 - n)/2$ equations, obtained from $t = (A,x)/(B,x)$ through elimination of t. The solution of this system alone involves already $n^5/24$ multiplication. A few hints concerning convergence acceleration for the conventional iteration method conclude the paper.

Courtesy of Mathematical Reviews E. Bodewig, Holland

1822. E. Doucet, On the resolution of a vector field into a sum of a curl and a gradient (in French), Proc. seventh int. Congr. appl. Mech. 2, Part II, 330–340 (1948).

To a given vector field u, determine a vector field v and a scalar field m such that u = curl v + grad m. By taking the divergence of both sides, m is found to satisfy m = div u. Once a particular solution m_1 has been constructed, by starting from the Newtonian potential, v is shown to be readily expressed by means of suitable integrations. A. van Wijngaarden, Holland

1823. H. Levy and E. A. Baggott, Numerical solutions of differential equations, New York, Dover Publications, Inc., 1950, viii + 238 pp. Cloth, 4.5 × 7 in., $3.

Book is a practical manual containing most of the familiar methods and many new ones. Chapters 1 and 2 are devoted to graphical integration with special attention given to singular points and the distribution of the integral curves. Chapter 3 contains the limited-range methods of Frobenius, Euler, Runge, Kutta with several modifications. Chapter 4 takes up forward integration and presents seven methods, starting with that of Adam-Bashforth. Chapter 5 discusses simultaneous and higher-order equations, and singles out five special types. Chapter 6 concentrates on linear equations of the second order, especially of the type $y'' + \rho(x)y = 0$ for several types of $\rho(x)$. Eigenvalue problems are not included. Checking procedures are well emphasized. Examples and exercises are plentiful. Ed.

1824. William Edmund Milne, Numerical calculus, Princeton, Princeton University Press, 1949, x + 393 pp. Cloth, 6.5 × 9 in., $3.75.

Book gives an excellent introduction into numerical analysis. From the large scope of this field that has developed with increasing speed during the last years, author has skillfully selected the most important topics:

I. Simultaneous linear equations (containing, among others, a method of solving linear equations and computing determinants by use of a calculating machine, based on the shortened Gauss' algorithmus); II. Solution of equations by successive approximations (iteration methods, also for the computation of eigenvalues of determinants); III. Interpolation (methods of Aitken, Neville, and Lagrange); IV. Numerical differentiation and integration (mean value formulas); V. Numerical solution of differential equations (mean value formulas only, the methods of Adams and Runge-Kutta are missing); VI. Finite differences (formulas of Newton, Gauss, Stirling, Everett, and Bessel); VII. Divided differences; VIII. Reciprocal differences; IX. Polynomial approximation by least squares (orthogonal polynomials, smoothing of data, Gauss' method of integration); X. Other approximations by least squares (trigonometric interpolation, Gram-Charlier approximation); XI. Simple difference equations. [Bracketed statements by the reviewer.] Three appendixes contain a list of notations and symbols, a bibliography, and a guide to several methods and formulas. A number of tables are added, for use of the computer.

Presentation is remarkably clear; it has been kept elementary "so that anyone with some knowledge of calculus and differential equations can read it understandingly."

F. A. Willers, Germany

1825. F. K. Rubbert, On numerical quadrature (in German), Z. angew. Math. Mech. 29, 186–188 (June 1949).

The remainder in numerical integration formulas has the general form $A(b - a)^m f^n(\xi)$, where A is the coefficient, $b - a$ is the interval of integration, and the nth derivative of the integrand function f is evaluated at a position (ξ) within the interval. It is possible to find pairs of formulas based on ordinates at differing positions, but with remainders for which m and n are the same, and with coefficients A differing and opposite in sign. Certain such formula pairs have been shown to bracket the true value of the integral, and the author gives pairs and examples for (m, n) equal to (5, 4), (7, 6), and (9, 8). He further points out that certain other pairs utilize the same ordinate position, but with remainders of differing orders of magnitude. Thus a simple subtraction yields a correction permitting rapid rounding off of results from the less precise formula. Vincent Salmon, USA

1826. C. C. Ritchie, Forward integration of differential equations, Proc. seventh int. Congr. appl. Mech. 4, 325–330 (1948).

1827. Gerard Fauré and J. Simon-Suisse, Two analog circuits for the inversion of symmetric matrices and determination of the critical velocity of flutter (in French), Proc. seventh int. Congr. appl. Mech. 4, 81–95 (1948).

Electric analog circuits for representation of inverted matrices and for a flutter system with two degrees of freedom are discussed. Properties of the circuit components along with their limits of accuracy are indicated. M. V. Barton, USA

1828. Z. V. Ryska, The course of temperatures in the combustion chamber and on the blades of gas turbines, Proc. seventh int. Congr. appl. Mech. 3, 226–237 (1948).

In a previous paper author described an electronic device for measurement of fluctuating temperatures based upon detection of infrared rays by a thermionic valve. Temperature changes are indicated both by a microammeter and by an oscillograph.

Author reports the application of the method to indication of temperatures in combustion chamber of a gas-turbine power plant. Irregularity of operation was greater than expected, and fluctuations were so rapid it was necessary to record photographically indications of both microammeter and oscillograph. Re-

sults were successful, however, and demonstrated need for changes in such things as fuel injection pressure, position of nozzles, direction of jet, and chamber shape. Paper is a short one, and does not make clear whether calibration has been established to convert the instrument indications into temperatures expressed in degrees. C. W. Smith, USA

1829. **M. J. Lighthill, A technique for rendering approximate solutions to physical problems uniformly valid,** Phil. Mag. 40, 1179–1201 (Dec. 1949).

Among the nonlinear equations of physics are those which, if some parameter α is assumed small, admit of a linearized approximation everywhere except in limited regions (e.g., shock waves of gas dynamics). Author's aim is to improve the approximation, making it uniformly valid with respect to α in these regions. Method is to expand the dependent and one independent variable of the equations in powers of α, introducing for the purpose a new variable, chosen with a view to suppressing undesirable singularities. Exposition is by successive examples, and the proofs, as author is careful to point out, are not always mathematically rigorous, but rather designed to convince physicists.

Paper is not for the casual reader—patient sifting of orders of magnitude of various terms is called for—but it should be of value to those seeking approximate solutions to the less tractable problems of compressible flow. W. J. Strang, Australia

1830. **R. H. J. Germay, On the resolution of systems of integral equations** (in French), Bull. Soc. Sci. Liége 17, no. 1, 2–5 (1948).

The usual method for solving systems of integral equations by successive approximation is modified slightly. Instead of replacing entire set of unknown functions $\phi_1(x), \phi_2(x), \ldots \phi_n(x)$ simultaneously at beginning of each iteration cycle, these are replaced individually as soon as a better approximation is found. This is possible since complete iteration cycle involves the solution of n equations, in the first of which an approximation for $\phi_1(x)$ is found; in the second an approximation for $\phi_2(x)$, etc. The suggested scheme is to use value obtained for $\phi_1(x)$ from first equation in the solution of $\phi_2(x)$ from second equation; again, to use $\phi_1(x)$, $\phi_2(x)$ thus obtained in solution of $\phi_3(x)$ from third equation, etc. The procedure is analogous to that used in Liebmann method for solving boundary-value problems by finite differences, where new improved values are immediately substituted for less accurate values without waiting until completion of an iteration cycle. A proof is contained for the absolute and uniform convergence of process, from which it follows that solutions so obtained satisfy the original system of integral equations.
H. Polachek, USA

1831. **R. H. J. Germay, On a method of successive approximations for the integration of systems of integral equations of Volterra's second class** (in French), Bull. Soc. Sci. Liége 17, no. 2, 26–30 (1948).

1832. **E. Ceressia, Method of successive approximations for the integration of linear systems of differential equations** (in French), Bull. Soc. Sci. Liége 16, no. 5, 84–93 (1947).

Two slightly different methods for the solution of systems of linear nonhomogeneous differential equations by successive approximations are obtained, both of which are modifications of the Picard method. Proofs are provided for the absolute and uniform convergence of the iteration process. The integration, instead of being a direct quadrature, now involves solution of a nonhomogeneous linear equation which can easily be carried out.
Ed.

1833. **M. V. Wilkes and W. Renwick, The EDSAC—an electronic calculating machine,** J. sci. Instrum. 26, 385–391 (Dec. 1949).

General description of arrangement and operation of digital computer recently completed at Cambridge, England. It is a serial machine, operating with binary numbers utilizing ultrasonic delay lines for storage. Total capacity of the store is 1024 numbers of 17 binary digits. Punched tape is used for input and a teleprinter for output. Stephen H. Crandall, USA

1834. **A. Bloch, Solution of algebraic equations by means of an electrolytic tank,** Proc. seventh int. Congr. appl. Mech. 4, 324 (1948).

1835. **J. Raleigh Nelson, Writing the technical report,** New York, London: McGraw-Hill Book Company, Inc., 1947, xiv + 388 pp. Cloth, 6.25 × 9.25 in., $3.50.

1836. **Tables of generalized sine—and cosine—integral functions: Part I, Part II. By the Staff of the Computation Laboratory, Harvard University,** Cambridge, Mass.: Harvard University Press; London: Oxford University Press; 1949, xxxviii + 462 pp.; 560 pp. Cloth, 6.9 × 9 in., $10 each part.

Mechanics (Dynamics, Statics, Kinematics)

1837. **Herbert Goldstein, Classical mechanics,** Cambridge, Mass.: Addison Wesley Press, Inc., 1950, vii + 399 pp. Cloth, 5.4 × 7.8 in., $6.50.

Book is intended for the modern physics student, and its basic idea is to use for classical mechanics the methods and concepts that serve as well in quantum mechanics, wave mechanics, and relativity. This means an emphasis on variational principles, canonical transformation, Hamilton-Jacobi and Poisson methods, and action-angle variables. Matrix treatment is favored and spinors are used in rigid-body theory.

Special relativity is presented early, and velocity-dependent forces are emphasized in examples. The latter are mostly quite different from conventional "museum pieces," and frequently derive from modern problems (as, e.g., rocket propulsion, scattering, etc.)

Continuous systems are treated by variational methods, and Lagrangian and Hamiltonian formulations are presented.

Chapters 1 and 3 review briefly and brilliantly the prerequisite intermediate mechanics.

Each chapter is followed by an annotated list of suggested additional reading. A list of the chapter headings follows: Survey of the elementary principles; Variational principles and Lagrange's equations; The two-body central force problem; The kinematics of rigid-body motions; The rigid-body equations of motion; Special relativity in classical mechanics; The Hamilton equations of motion; Canonical transformations; Hamilton-Jacobi theory; Small oscillations; Introduction to the Lagrangian and Hamiltonian formulations for continuous systems and fields.
A. W. Wundheiler, USA

1838. **R. Gran Olsson, Remarks on the motion of anchor chains,** Quart. appl. Math. 7, 463–470 (Jan. 1950).

Author considers problem of a weight sliding down an inclined plane without friction while at same time dragging a chain which is fed out from a coil at top of plane. Problem has been treated by Timoshenko and Young (S. Timoshenko and D. H. Young, *Advanced Dynamics*, McGraw-Hill Book Co., New York, 1948, pp. 114–116) who gave an expression for velocity of weight

as a function of distance traveled. By defining a parameter, u, which is an elliptic function of weight, linear density of chain, and distance traveled, author has succeeded in deriving expressions for: time of travel, distance traveled, velocity, acceleration, momentum, kinetic energy, potential energy, and loss of energy through nonelastic collision of links, all involving Weierstress functions $p(u)$ and $\zeta(u)$. Hence, a complete description of kinematics of the system can be expressed in terms of a single parameter, u. Horace M. Trent, USA

1839. I. G. Malkin, The methods of Liapounoff and Poincaré in the theory of nonlinear vibrations (Metodí Lyapunovai Puankare v teorii nelineĭnikh kolebaniĭ) (in Russian), Leningrad-Moscow, OGIZ, 1949, 243 pp. Paper, 4.2 × 6.5 in., $1.

Book culminates in presentation of author's own results on systems close to Liapounoff's (REV 3, **1036**). Chapter titles are: General Poincaré theory of periodic motions, 24 pp.; Oscillations of a quasilinear system, 65 pp. (for a quasilinear system the coefficients of the expansion in powers of the small parameters and initial values satisfy a system of linear equations with constant coefficients); Stability of periodic motions, 48 pp.; Liapounoff's theory of periodic solutions, 27 pp.; Oscillations of systems of one degree of freedom close to Liapounoff's systems, 38 pp.; Oscillations of systems of several degrees of freedom close to Liapounoff's systems. A. W. Wundheiler, USA

1840. E. C. Pike, Coefficients of friction, J. roy. aero. Soc. 53, 1085–1094 (Dec. 1949).

Paper summarizes available data on coefficient of friction between rubber tires and various types of roadway. Numerical data are included but are considered approximate because of inherent variation in road texture, and because objective in many of tests was primarily to distinguish between good and bad roads. Speeds are limited to 40 mph. Charles E. Crede, USA

1841. P. H. Thomas, Surface temperatures due to friction between sliding solids, Proc. seventh int. Congr. appl. Mech. 3, 300–301 (1948).

Abstract of unpublished paper.

Gyroscopics, Governors, Servos

(See Revs. 2024, 2029, 2064)

Vibrations, Balancing

(See also Revs. 1839, 1945)

1842. Miguel Hacar, Vibrations in solids and elasticity (in Spanish), Info. Constr. 2, 12 pp. (Jan. 1950).

In this very interesting didactical paper author begins his discussion by taking the classical equations that govern vibrations of an elastic solid and demonstrates that a solution of these equations may be effected in terms of two vectors: irrotational, and solenoidal. These vectors are associated with longitudinal and transverse waves. Analogies are then drawn between the dilatation of a solid and its temperature in the steady state, and transverse waves are compared with electromagnetic waves.

Paper concludes with a discussion of topics in higher mathematics that should be included in the curriculum of a research engineer or scientist, and with a discussion of vibration problems in engineering structures. Louis A. Pipes, USA

1843. L. M. Milne-Thomson, The pendulum, Quart. J. Mech. appl. Math. II, 479–480 (Dec. 1949.)

Concise treatment of simple pendulum by elliptic functions. Charles E. Crede, USA

1844. G. D. McCann, Jr., and R. R. Bennett, Vibration of multifrequency systems during acceleration through critical speeds, J. appl. Mech. 16, 375–382 (Dec. 1949).

This experimental study of torsionally vibrating systems was conducted on Cal Tech's electric-analog computer, using force-torque analogy. The actual physical system comprised an induction motor coupled to two loads, as generators. On starting, slip-frequency vibration changes from line frequency to practically zero. Hence forcing function in analog computer was supplied by an oscillator whose frequency could be changed smoothly from a large value to zero, or vice versa. Results were obtained in form of oscillograms corresponding to shaft torque as a function of time; in extensive graphical presentations, the maximum torques in shafts were plotted against a variable involving rate of change of frequency, with ratios of shaft stiffnesses and rotor moments of inertia as principal parameters. It was found that shaft torque maxima were not reached until some time after driving torque had passed through resonance. Also, crest torque for decreasing frequency was in most cases 10 per cent to 15 per cent above that for increasing frequency. Curves are plotted for increasing frequency, so for use in problems of motor-generator starting, the plotted torques should be increased about 15 per cent to give actual torque. Vincent Salmon, USA

1845. O. Emersleben, The period of a revolving pendulum as an analog to the potential of a circle (in German), Z. angew. Math. Mech. 29, no. 9, 279–282 (1949).

In terms of suitable parameters and coordinates, the period of a pendulum and the potential in a plane due to a charged ring each lead to an expression of the form $(a/x)K(b/x)$, and so have a formal analogy.

Courtesy of Mathematical Reviews P. Franklin, USA

1846. Rune L. Evaldson, Robert S. Ayre, and Lydik S. Jacobsen, Response of an elastically non-linear system to transient disturbances, J. Franklin Inst. 248, 473–494 (Dec. 1949).

Authors present an analysis of transient response caused by motion of spring support of an undamped, nonlinear, one degree of freedom system. A nonlinear spring has a stiffness k_1 for $|x| < G$ and a stiffness k_2 for $|x| > G$, where x is the relative displacement between mass and support. Three kinds of pulses are considered for displacement of support, viz., rectangular, cosine, and "skewed" cosine. Maximum relative displacement is given graphically in dimensionless form as a function of time of pulse duration, with X_g/G and k_2/k_1 as parameters, where X_g is maximum displacement of support. Maximum relative displacement may occur either during or after displacement pulse. It is noted in paper that simple and general quantitative conclusions cannot be drawn. Results were obtained graphically and experimentally. Graphical procedure develops trajectories in a modified phase plane. Albert I. Bellin, USA

1847. J. M. Frankland, Effects of impact on simple elastic structures, Proc. Soc. exp. Stress Anal. 6, no. 2, 7–27 (1948).

A number of representative cases of impact loading and their effects upon a simple undamped system with one degree of freedom are discussed. For nonoscillating disturbances, effects are governed mainly by abruptness of original application and by duration of disturbance in terms of natural period of vibration of

the system. For oscillating disturbances, there is an added influence similar to resonance in steady-state condition.

Applications to design of structures under impact are discussed and suggestions are made for basis of design of decks, superstructure, and gun foundations under blast and gun recoil loads. Application to design of instruments for making observations during impact tests and to interpretation of their records is considered briefly. A formal mathematical treatment of subject is presented in the appendixes. Nicholas Di Pinto, USA

1848. **S. Woinowsky-Krieger, On vibrations of a two-bar elastic system with a small rise,** J. appl. Mech. 16, 395–398 (Dec. 1949).

A hinged-hinged "curved" beam is formed by joining two straight bars by a center hinge, the system having a very small ratio of rise to span. A transverse force is applied at center hinge; the static load-deflection curve is found to be nonlinear and to exhibit a maximum at the "breaking-through" load. The system is idealized to one of a single degree of freedom, and the free vibrations of beam are written in terms of elliptic functions. Elastic stability of system under a constant load, suddenly applied, is discussed; critical or breaking-through load is found to be less by 23 per cent than for static case. Author points out that "The system treated is the simplest one in a class of systems, whose elastic stability under statical loads has been investigated in some detail, but whose dynamical properties are still known only very incompletely." Robert D. Specht, USA

1849. **R. L. Sutherland, Bending vibration of a rotating blade vibrating in the plane of rotation,** J. appl. Mech. 16, 389–394 (Dec. 1949).

Report describes application of a general iterative numerical method, originally developed by Myklestad, to specific problem of determining uncoupled natural vibration modes and frequencies of a rotating blade for those vibrations that occur in plane of rotation. Numerical examples of method are given. It is shown for corresponding blades that natural frequency of bending vibrations in plane of rotation is somewhat lower than natural frequency for vibrations perpendicular to plane of rotation, since in former case, centrifugal forces are not as effective in raising bending frequency as in latter case. Benjamin Smilg, USA

1850. **D. M. Smith, Vibration of turbine in packets,** Proc. seventh int. Congr. appl. Mech. 3, 178–192 (1948).

Calculations are made of natural frequencies and modes of vibration of a packet of parallel blades of uniform section, with blade tips connected by a single shrouding strip attached to each blade. For a packet of n blades, modes of vibration fall into groups: (1) one fundamental mode, with blades in phase; (2) a narrow band of $(n - 1)$ modes with blades not in phase; (3) one harmonic with blades in phase, and so on. Packet frequency is shown to depend upon fundamental frequency of a blade not shrouded, mode of vibration, and two ratios (of mass and rigidity) relating shrouding characteristics to blade characteristics.

It is assumed that blades are identical, parallel, uniform, and rigidly fixed at their roots. Shrouding is taken as uniform, rigidly attached to each blade at its tip, and maintaining constant pitch between successive blades in small flexural vibration. Only movement in plane of packet is considered. From a consideration of moments and forces acting on ends of blades, and deflection equation for a uniform cantilever, the determinantal equation is derived. Problem is solved for packets of 6 and 20 blades, respectively.

A discussion of validity of the physical assumptions is made from viewpoint of extending method and results to practical applications.

Consideration is also given to effect of introduction of a lacing wire into shrouded packet. For modes where blades are not vibrating in phase, vibrations can be suppressed if lacing wire is introduced at a suitable height. Selwyn Gendler, USA

Wave Motion, Impact

(See also Rev. 2027)

1851. **Albert E. Heins, Water waves over a channel of finite depth with a submerged plane barrier,** Canad. J. Math. II, no. 2, 210–222 (1950).

Velocity potential is determined which governs surface waves past a semi-infinite plane barrier placed below undisturbed free surface level and parallel to channel bottom. Fluid is assumed incompressible, nonviscous, and irrotational. Reflection and transmission coefficients for barrier are obtained in explicit form. Method uses Green's functions, Fourier transforms, and an integral equation of Weiner-Hopf type. Third paper in a series. S. A. Schaaf, USA

1852. **F. Ursell, Surface waves on deep water in the presence of a submerged circular cylinder. I, II,** Proc. Camb. phil. Soc. 46, 141–158 (Jan. 1950).

A method is given for calculation of velocity potential for surface waves of small amplitude generated on deep water by a periodic normal velocity distribution prescribed over a submerged circular cylinder. It is shown that the reflection coefficient of waves incident on a fixed circular cylinder vanishes, as was shown by Dean [Proc. Camb. phil. Soc. 44, p. 483 (1948)]. Pulsations of a submerged cylinder are discussed for particular case of a normal velocity the same at all points of cylinder at any given time. Method of solution involves a system of linear equations in an infinite number of unknowns. Unknowns are obtained either in form of a power series convergent for sufficiently small values of a parameter related to cylinder radius, or in terms of convergent infinite determinants. In part II a uniqueness theorem is established. S. A. Schaaf, USA

1853. **R. E. Blake, Peak-reading accelerometers,** Nav. Res. Lab. Rep. 5-3276, 45–53 (1948).

Mechanical gages designed to indicate maximum acceleration of shock motion are discussed. These are: (a) putty gage, (b) Buchanan gage, a weighted-end cantilever of same type as putty gage, (c) indenter gage, and (d) ball-crusher gage. Principle of operation of each gage is outlined. Sources of error are discussed, particularly those due to duration of acceleration. Necessary conditions for minimizing these errors are pointed out, and work yet to be done indicated. From author's summary

1854. **J. W. Grier, Multifrequency reed gages, types 1 and 2,** Nav. Res. Lab. Rep. 5-3276, 32–44 (1948).

Performance characteristics of multifrequency reed gages, as well as their theory and applications, are presented. Author's summary

1855. **E. W. Krammer, Strain gage and carrier-type accelerometers,** Nav. Res. Lab. Rep. 5-3276, 61–72 (1948).

1856. **W. H. Hoppmann, 2nd, Impact of a mass on a column,** J. appl. Mech. 16, 370–374 (Dec. 1949).

Problem of motion of a uniform bar subjected at its midpoint

to a transverse impact is considered in this paper. Bar is assumed to be simply supported at its ends and spring supported against transverse motion by a continuum of identical massless springs distributed along its length. It is subjected to constant axial end forces. Pressure exerted by striking mass is taken to be a sinusoidal pulse whose magnitude and duration are adjusted by consideration of Hertz contact theory. Readers should consult a previous paper [J. appl. Mech. 15 (1948), p. 125] by the author for a discussion of basis for this choice. Neglecting damping, author then solves the differential equation governing motion of bar. Three numerical examples are worked out in detail.

L. E. Goodman, USA

1857. Alexander Kromm, On the propagation of shock waves in discs with a circular hole (in German), Z. angew. Math. Mech. 28: no. 4, 104–114; no. 10, 297–303 (1948).

Problem discussed is propagation of two-dimensional elastic waves in an infinitely thin plane disk of infinite extent. Two cases are investigated: (*a*) a constant radial pressure is applied instantaneously at $t = 0$ and sustained until $t = \infty$; (*b*) a uniform radial velocity is imparted to all points on the perimeter.

Solution is obtained by means of Laplace transform and leads to an integral equation of Volterra type of first kind; this equation is solved numerically, assuming Poisson's ratio, σ, of material 0.3. Results are shown in curves giving variation of particle velocity, radial stress and hoop stress with t and radial distance, r, measured from center of hole.

The disturbance is propagated, without dispersion, with velocity $c = [E/\rho(1 - \sigma^2)]^{1/2}$. Discontinuities in particle velocity and in stresses at wave front are proportional to nondimensional variable $(a/r)^{1/2}$, and value of ratio of particle velocity to radial stress at wave front is $[E\rho/(1 - \sigma^2)]^{1/2}$. Behind wave front, disk ultimately attains a steady state and solutions are shown to converge to values corresponding to static cases.

Particular interest of paper lies in method of calculation and in numerical results. It forms an excellent starting point for solving the more important problem of a disk of finite thickness, where one would expect dispersion to occur, infinitely long waves traveling with the velocity c, and infinitely short waves degenerating into Rayleigh surface waves.

R. M. Davies, Wales

Elasticity Theory

(See also Revs. 1842, 1857, 1867, 1871, 1875, 1876, 1917, 1940, 1962, 1966)

1858. A. E. Green and I. N. Sneddon, The distribution of stress in the neighbourhood of a flat elliptical crack in an elastic solid, Proc. Camb. phil. Soc. 46, 159–163 (Jan. 1950).

Paper is concerned with distribution of stress near a flat elliptical crack in a body of infinite extent under a uniform tension at infinity perpendicular to crack. This problem can be regarded as a special case of a more general one which has been treated by Sadowsky and Sternberg [J. appl. Mech. 16, p. 149 (1949); REV 3, **440**] but the method of solution adopted by present writers is somewhat different from the more general work of mentioned authors.

Main characteristic of treatise consists in that problem is reduced to a hydrodynamical problem, the solution of which is well known: viz., determination of velocity potential due to a flat elliptical disk moving with uniform velocity perpendicular to its plane through an infinite incompressible fluid, which is at rest at infinity.

The problem of indentation of a flat semi-infinite surface by a flat-ended elliptical cylindrical punch is also considered, but only partly solved. It finds its analogon in the velocity-potential problem of perfect fluid flow through an elliptical aperture in a thin rigid body.

The statement that the given solutions are "surprisingly simple" will be confirmed by only those who have the disposal of adequate mathematical knowledge.

C. B. Biezeno, Holland

1859. M. T. Huber, Theory of elasticity (**Teoria sprezystosci, Vol. 2,** Krakow, Polish Acad. Sci., 1950, xii + 360 pp., 65 figs. Paper, 7 × 10 in. Price unstated.

This second volume contains new material of research value. After a discussion of equilibrium and motion of an unbounded elastic body, of equilibrium of axisymmetric bodies, and of H. Neuber's stress function and its applications, author presents theory of isotropic plates. Contained here is a revised solution of the problem of an unbounded floating plate under a concentrated load; the revision results in a correction of a numerical value computed by H. Hertz and reproduced in *Handbuch der Physik*, vol. 6.

This is followed by a presentation of theory of orthotropic plates. The next chapter deals with shell theory.

In last chapter, devoted to foundations of classical plasticity theory, author subjects Mohr's yield condition to detailed criticism and promotes his own condition, expressed in terms of ultimate value of pure elastic shape distortion. He further takes issue with Roš-Eichinger interpretation of his work as a generalized and improved Mohr's theory. Mohr's condition does not allow for influence of intermediate principal stress, at variance with Lode's and later experimental work. Book concludes with an account of author's work on self-hardening of gun barrels.

The book is remarkable for the completeness and rigor of its arguments.

J. Nowinski, Poland

1860. G. Colonnetti, Elastic equilibrium in the presence of permanent set, Quart. appl. Math. 7, 353–362 (Jan. 1950).

Author considers first a theory of elastic-plastic equilibrium based on the following assumptions: (1) There is a one-to-one correspondence between stresses and strains in each element below elastic limit; (2) as soon as limit is reached, deformation can increase indefinitely under constant stress. When loads applied to body are removed, plastic or permanent strains result. In general, these are not compatible and a state of residual stress arises. A certain portion of the work performed is not recovered but remains as "residual" energy. Presence of these residual stresses explains the apparent rise in elastic limit when body is reloaded as well as Bauschinger effect. It is also pointed out that residual stresses may arise in a structure in which creep is present.

Mechanical work required by such nonelastic deformations consists of two parts: one is spent to produce the deformation itself and is lost through irreversible processes; the other gives rise to state of residual stress. Let ϕ denote the total elastic strain energy of the body; $\bar{\epsilon}_x, \ldots, \bar{\gamma}_{xy}$ the plastic strains; and $\sigma_x, \ldots, \tau_{xy}$ the stresses. The author shows that for a given system of loads and nonelastic strains, the stresses characterizing equilibrium render $\phi + {}_v\int(\bar{\epsilon}_x\sigma_x + \ldots + \bar{\gamma}_{xy}\tau_{xy})dv$ a relative minimum. Variations in the stresses considered are assumed not to alter nonelastic strains. Author predicts the possibility of establishing a new theory of strength of beams on basis of this principle.

G. H. Handelman, USA

1861. Tadasi Isibasi, Form-factors of solids of revolution in relation to those of plane plates (in Japanese), Trans. Soc. mech. Engrs. Japan 14, no. 47, part 1, 86–89 (Oct. 1948).

Paper shows method of finding form-factors α_n in an elastic body of revolution having a comparatively shallow notch and sub-

jected to an axial load, from the form-factor α_e in a plate having a profile of same form as meridian section of former body and subjected to similar load. Neglecting some insignificant factors from respective equations of compatibility which have to be satisfied by Airy's stress function in a plate, and Weber's stress function F_w in a body of revolution, an approximate relation between F_e and F_w is deduced, which may hold in the neighborhood of a free surface. Assuming that points of maximum stress approximately agree in both bodies, and using the respective free boundary conditions, an approximate relation between α_e and α_w is deduced in a form containing two coefficients, which are determined by using Neuber's graph about a hyperbolic notch.

Teruyoshi Udoguchi, Japan

1862. **O. Föppl, Plastic and elastic deformations connected with surface pressures (Die mit dem Oberflächendrücken verbundenen plastischen und elastischen Formänderungen),** Braunschweig, Mitt. Wöhler-Inst. Friedr. Vieweg & Sohn, no. 42, 1949, 66 pp. Paper, 6 × 8 in., DM 4.

An approximate method is presented for calculation of residual stresses in cylinders in which surface layers have undergone plastic flow. Yielded surface layers are first assumed to deform independently, and continuity in displacement between these layers and remaining elastic parts of cylinders is then restored by application of an axial and radial system of (residual) stresses. Results of calculation are found to be essentially in agreement with available experimental data. M. Hetényi, USA

1863. **O. Föppl, The natural elasticity constants and the specified (natural) shear stress direction (Die natürlichen Elastizitätskonstanten und die ausgezeichnete (natürlichen) Schubspannungsrichtung),** Braunschweig, Mitt. Wöhler-Inst. Friedr. Vieweg & Sohn, no. 43, 1949, 20 pp. Paper, 6 × 8 in., DM 1.60.

Author proposes that, instead of Young's modulus and Poisson's ratio, elastic behavior of a material may be more conveniently characterized by its "natural" elastic constants, namely, by the modulus of volume change (bulk modulus) and the shear modulus, which two can be determined independently. Natural shearing stress directions are defined by three mutually perpendicular planes in which state of stress is same. It is shown that any general state of stress may be resolved into a hydrostatic normal stress and two such "natural" shearing stress systems.

M. Hetényi, USA

Experimental Stress Analysis

(See also Revs. 1879, 1943)

1864. **E. Perrin, Adaptation to field use of deformation-recording apparatus** (in French), Inst. tech. Bâtim. Trav. Publ. no. 126 (Mar.-Apr. 1950).

Author describes some simple modifications introduced in laboratory mechanical and optical fleximeters and strain gages to adapt them to field use, and claims that the instruments are sturdy, precise, and inexpensive. The most remarkable point of the paper seems to be the recording optical fleximeter which can be used in daylight. An example of application to a vibration problem is also given. A. J. Durelli, USA

1865. **M. Hetényi, The present state of development of experimental stress analysis,** Proc. seventh int. Congr. appl. Mech., 57–75 (Sept. 1948).

An interesting summary of methods used in stress analysis is presented, together with side lights drawn from personal experience. Subjects described include mechanisms of failure, interferometry, brittle lacquers, and two- and three-dimensional photoelasticity. Paper has an extensive bibliography.

A. J. Meyer, USA

1866. **E. H. T. Jackson, An electronic method for the measurement of an electrical quantity and its application to the indication of strain,** Proc. seventh int. Congr. appl. Mech. 4, 128–136 (1948).

An electronic measuring instrument of large effective scale length is described. Instrument, adaptable to measurement of strain in conjunction with resistance-wire strain gages, is composed of form units: (1) Wheatstone bridge network; (2) d-c amplifier: (3) pulse generator; (4) display unit.

Magnitude of bridge output is indicated by a time delay in production of an electrical pulse of short duration. For zero bridge output, electric pulse occurs at beginning of timing cycle. As bridge output increases, pulse is advanced in time. Author describes the use of four CRO tubes to indicate bridge output to four significant figures. Instrument, as described, is said to have a sensitivity of 1 part in 100,000. George H. Lee, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1881, 1908, 1915, 1928, 1935, 1948)

1867. **A. M. Guzman and C. J. Luisoni, On the simply supported short beam** (in Spanish), Cienc. Tecn. 114, no. 576, 285–292 (June 1950).

One of a series of articles dealing with stress distribution in short beams (Revs 1: **57, 1608; 2, 172; 3, 665**). Authors studied case of beam of height equal to span simply supported and uniformly loaded on lower boundary, by using Galerkin's method. After Sciammarella and Palacio verified their results photoelastically, authors here extend the method to other loading conditions and other widths of supports. They do this by changing the coefficients of the Airy function used. At the end of the paper, authors raise the question of accuracy and convergence of Galerkin's method and point out that for a beam of height equal to 1.5 times the span, results seem to deviate from exact results.

A. J. Durelli, USA

1868. **G. Winter, W. Lansing, and R. B. McCalley, Performance of laterally loaded channel beams,** Eng. Struct., N. Y., Academic Press, 49–60 (1949).

Paper is concerned with determining spacing and strength of braces required to counteract twisting tendency of channels loaded by forces in plane of the web. Theory of elastic behavior of eccentrically loaded channels is discussed briefly, using as a basis the work of Timoshenko and Goodier. Experimental stress data are given showing effect of brace location. These data show that local maximum stress does not govern strength, since plastic stress redistribution allows understressed portions to carry additional load. Taking this action into account, paper shows that maximum efficiency can be attained with a relatively small number of braces. R. L. Bisplinghoff, USA

1869. **B. E. Gatewood, Shear distribution in beams with variable webs,** J. aero. Sci. 16, 749–753 (Dec. 1949).

A rational method for analysis of deep cantilever beams is developed. Considered beams are uniplanar, built up of a thin web of variable thickness divided into longitudinal panels by parallel longitudinal stringers. It is assumed that stringers carry only direct stress, webs only shear, and that the latter is uniform, in a given panel, over cross section. With these assumptions, the

analysis is shown to be governed, in case of n panels, by $n - 1$ second-order linear differential equations which, together with the requirement of statical equilibrium and boundary conditions, determine shear distribution. Solution of these equations is worked out in detail for case of a three-panel beam with identical stringers, and results are presented graphically in terms of two dimensionless parameters, ratio of outer to center web thickness p, and

$$h = l(t/Ah)^{1/2}$$

where l is beam length, t outer web thickness, A stringer area, h panel height. These conclusions are supported by tests of a three-panel beam carrying a concentrated load.

L. E. Goodman, USA

1870. Anton Pomp and Max Hempel. Frequency of breakage and surface condition of coil springs (in German), Arch. Eisenhuettenwesen 20, no. 11/12, 385–393 (Nov./Dec. 1949). [Paper no. 505 of the Max-Planck-Institute for Iron Research.]

Influence of processes of manufacture on frequency of breakage of coil springs, made from steel wire of following analysis was investigated: carbon, 67, silicon, 20, manganese, 51, phosphorous, .016, sulphur, .019.

Bars from same heat were rolled in three different steel mills. Bars were rolled to rounds of 6.75 mm and then drawn to wires of 3.9-mm diam, quenched in oil at 1432 F and drawn in lead at 932 F to a tensile stress of 200,000 to 227,000 psi.

After heat treatment, wire was ground to a diameter of 3.6 mm and wound to springs of 8 turns and an outside diameter of 32 mm.

The springs were all compressed over a stroke of 20 mm from an installed height of 48 to 51 mm, in groups of 12 springs, up to 4 groups being tested simultaneously in same testing machine at a speed of 1500 compressions per min. Well over 1000 springs were compressed up to 10 to 12 million times, some up to 14 to 16 million, and a very few up to 60 million times. Torsional stresses were computed according to the Wahl formula.

Test results are recorded in tables and diagrams. Material stress varied from initial value of about 32,000 psi to maximum value of from 92,000 to 120,000 psi. Following conclusions were drawn from test results: (1) Fatigue and transversal breaks may occur on one or two turns of same spring. (2) In case of fatigue breaks with a smooth edge, break spreads along one front; if edge is offset, break spreads along two fronts. (3) Majority of breaks occur at inside of turns, where stress is greatest, and mostly in third to fifth turn. (4) Dispersion in number of compressions in individual groups, mainly due to different surface conditions, varies between 1 and 10 million. (5) Differences in free length of springs increase dispersion in number of compressions before breakage, and should be considered. (6) Quality of grinding and surface irregularities of wire strongly affect life of the springs, while faulty rolling, drawing, and winding affect it very slightly.

O. R. Wikander, USA

1871. H. Okubo, On the torsion of a prismatic cylinder with a star-shaped section, J. appl. Phys. 20, 1155–1157 (Dec. 1949).

A solution of Saint Venant's torsion problem is given for a prismatic cylinder with an arbitrary number of identical notches. The bounding curve is asumed to be composed of logarithmic spirals. Comparison is made between calculated and measured stiffness of a test shaft with 18 notches, and good agreement is obtained.

C. C. Wan, USA

1872. M. Rousseau, On the determination of the maximum sag of an aerial electric line (in French), Hommage Faculty appl. Sci., 186–189 (1947).

Author considers deflections produced in aerial electric cables by temperature, ice, and wind. Parabolic approximation to equation of cable is used, and formulas describing effects of ice and wind are stated.

Ed.

1873. P. G. Hodge, Jr., On torsion of plastic bars, J. appl. Mech. 16, 399–405 (Dec. 1949).

Analysis is based on conventional definitions of stress, strain, and stress function. Plastic yielding is assumed to take place when seam of squares of orthogonal shearing stress components equals a constant, and the ratio of one shearing stress component to the accompanying strain is assumed equal to corresponding ratio for other component.

A general equation is developed for warping of cross section, but this contains a warping function that is difficult to obtain. One suggested solution consists in treating plastic state as limiting condition of elastic-plastic state, in which case, elastic core of cross section reduces to a region, the boundaries of which may be approximated by a mathematically simple curve. From this, the warping function may be readily obtained. Method is illustrated in its application to a rod with an oval cross section, and to an I-beam.

Consideration is also given to residual stresses developed by plastic torsion. A general procedure is established for analysis and two examples are worked to illustrate application of the method.

Glenn Murphy, USA

Plates, Disks, Shells, Membranes

(See also Rev. 1857)

1874. H. G. Hopkins, Elastic deformations of infinite strips, Proc. Camb. phil. Soc. 46, 164–181 (Jan. 1950).

Fourier integrals are used to solve some problems of generalized plane stress and small transverse displacements in infinitely long rectangular plates stressed only at edges. Four general types of problems are solved, i.e., case when stresses are prescribed along free edge, case when displacements in plane of plate are prescribed along free edge, case when bending moments and total transverse shear forces are prescribed along free edge, and case when transverse displacement is prescribed along free edge. Paper concludes with a discussion of problem of a concentrated force on edge.

R. M. Wingren, USA

1875. S. J. Dokos, A force applied in the median plane at the center of a circular insert in a plate, J. appl. Mech. 16, 411–413 (Dec. 1949).

Problem concerns determination of state of plane stress in an infinite plate containing a cylindrical hole filled with a material having elastic properties which differ from those of plate. A concentrated load acts in median plane of plate and at center of circular inclusion. Upon the stress system due to a point force in an infinite plate [Timoshenko, *Theory of Elasticity*, 1934, p. 109], is superimposed a suitably chosen stress system which does not violate loading conditions, vanishes at infinity, and maintains continuity conditions at junction of inclusion. Results include case of a rigid inclusion.

D. L. Holl, USA

1876. A. Holmberg, Theory of elasticity for circular plates with special reference to cylindrical aelotropy (in Swedish), Bygnstat. Medd. 19, no. 3, 75–92 (1948).

Author studies design of reinforced concrete circular plates by studying solution of Lagrange plate equation for case of cylindrical aelotropy and Poisson's ratio zero. The ratio n of the material constants D_r and D_θ is chosen such that $0.5 < n < 1$ since it gives

a good moment distribution. The value $n = 9/16$ gives very convenient design formulas. Graphs display moment variations under uniform normal loading and central concentrated loads for different edge conditions. D. L. Holl, USA

1877. **H. Okubo, Bending of a thin circular plate of an aeolotropic material under uniform lateral load (supported edge)**, J. appl. Phys. 20, 1151–1154 (Dec. 1949).

Plate is assumed to have three planes of symmetry in regard to its elastic properties, resulting in four elastic constants for material of plate instead of two, as for an isotropic material. Deflection of plate resulting from a uniform lateral load is expressed in form of a polynominal plus two infinite series with undetermined coefficients. If series converges properly, very little error is introduced by neglecting all but first few terms in each series. The requirements of zero deflection and zero normal moment at boundary provide sufficient equations for determination of the unknown coefficients.

A numerical example is given and from results, author proposes empirical relations which in effect express an equivalent Young's modulus and an equivalent Poisson's ratio in terms of four assumed elastic constants. He then suggests using these two equivalent constants in usual formulas for bending of circular plates. Gerald Pickett, USA

1878. **B. C. Carter, J. C. Ghosh, M. V. C. Sastri, and K. V. Chinnappa, Measurement of surface strains in diaphragms,** Engineering 168, no. 4375, 581–583 (Dec. 1949).

In using metal diaphragms in pressure gages, pressure is usually obtained from measurement of central deflection of plate. This method has good sensitivity with moderate pressures. As maximum pressure is increased, thickness of diaphragm has to be greater in relation to diameter and maximum deflection decreases. Thus it becomes difficult to measure deflection with sufficient accuracy; on the other hand, however, surface strain can still be accurately measured by means of electrical strain gages. Paper reports the results of an experiment to determine the proportionality factor between strain-pressure. Experimental values are checked with theoretical results and maximum deviation is found to be 8 per cent. C. T. Wang, USA

1879. **K. E. Knudsen, Wm. H. Munse, and B. G. Johnston, Stresses in hot metal ladles,** Iron Steel Engr. 26, 49–70 (Dec. 1949).

By a hot-metal ladle, authors mean an open vessel reinforced by rings, such as used in steel industry to carry molten metal. Paper gives results of experiments on $^1/_5$ scale models of 150-ton capacity prototypes, and suggests a method of stress analysis. Three types of ladles were tested. Type A is a round riveted ladle, type B an oval welded ladle, type C a round welded ladle with dished bottom. Strains and deflections were measured. Temperature stresses were neglected.

It was found that the general structural behavior of ladle models deviated considerably from one usually assumed in design. Main results are: Cross section has a tendency to become oval under loading, the trunnions moving in and the region between trunnions moving out. Deflections increase nearly linearly from bottom up. Bottom plate behaves as an elastically supported circular plate.

Stresses in stiffening rings are primarily bending stresses, and reach maxima at trunnions and midway between trunnions. Normal stresses on side shell are considerably smaller than the ring stresses, except near bottom, where a stress concentration occurs. This general description of behavior holds for all types of ladles. Numerous recommendations are given as to material, proportions, type of construction, design assumptions, and methods of analysis. Formulas to be used in analyzing are given. Reference for their derivation is made to a dissertation written by K. E. Knudsen at Lehigh University. Bruno A. Boley, USA

1880. **A. G. Pugsley, On a strut in a non-linear medium and the waves in a tension field shear web,** Eng. Struct., N. Y., Academic Press, 111–119 (1949).

Experiments have indicated that wave length of buckles in a tension field web decreases with increasing shear load. In order to answer the question as to what determines wave length and what causes observed change, author develops a theory based upon hypothesis that a diagonal compressive strip in a tension field can be regarded as an elastic strut in a nonlinear medium.

Analysis is given for a long uniform strut subjected to axial compression and restrained against bowing laterally by a continuous elastic foundation such that resistance provided to a lateral displacement y is of type $(ay + by^3)$ per unit length of strut. It is then shown that behavior of a strip of a shear web running diagonally in direction of compressive stress is analogous to that of strut. Elastic restraint is provided by tautness of tension field which is perpendicular to axis of strip, and it is shown that this elastic restraint has same form as assumed in strut analysis.

Application of this theory leads not only to a formula for wave length of buckles, but also to a simple relation between tensile and compressive stresses in a buckled shear web. Theoretical results check reasonably well with experimental data obtained by Crowther and Hopkins. Dana Young, USA

1881. **J. P. O. Silberstein, J. R. M. Radok, and H. A. Wills, The strength of wooden box beams,** Proc. seventh int. Congr. appl. Mech. I, 238–353 (1948).

This is a theory of flexure of wood box beams developed by assuming a peculiar form for stress-strain relation of wood in compression (i.e., for stress distribution across cross section of beam). Formulas obtained contain three unknown coefficients, the values of which are to be found by experiment.

Authors tested over 200 beams of hoop pine and mountain ash; they give experimental values of coefficients referred to above in form of diagrams. A numerical example is given to illustrate method. Ch. Massonnet, Belgium

1882. **T. H. Gawain and E. C. Curry, Stresses in laterally loaded disks of nonuniform thickness,** Prod. Engng. 20, 130–134 (Dec. 1949).

Authors consider a circular disk of nonuniform thickness, which may be solid or have a center hole. Loading is transverse and has radial symmetry.

The disk is approximated by a number of elements, each of which is a hollow disk of uniform thickness. Loading is approximated so that shearing force over each element varies inversely as radial distance.

Using classical theory of symmetric bending of circular plates of uniform thickness (S. Timoshenko, *Theory of Plates and Shells*, McGraw-Hill, 1940, pp. 55–58), authors compute increments in bending moments over each element. Using this result, a scheme is then set up for computation of bending moments at outer edge of original disk in terms of bending moments at inner edge. A second similar scheme is set for case of a certain particular type of loading. Superposition of results of these two computations then permits the satisfaction of actual boundary conditions on both inner and outer boundaries of original disk. G. E. Hay, USA

Buckling Problems

(See also Revs. 1856, 1880, 1906, 1924, 1925, 1926)

1883. H. L. Cox, Effective stiffness of stringers to locally applied loads, Eng. Struct., N. Y., Academic Press, 165–178 (1949).

Paper presents analysis of influence of shear lag, deformation of cross section of stringer, and edgewise compression on length of the half wave of buckling λ and on foundation modulus K of stringers which stiffen thin sheet structures. Role of stringers in resisting local deformations is noted, and importance of general theory of elasticity for the problem is stressed. However, mathematical theory of elasticity is not used.

Theoretically determined stiffness factors for Z-section stringers are presented in a table. Experimentally determined load-deflection data for a very short section of Z-stringer loaded at points along one flange with concentrated loads normal to the flange are given in form of curves. W. H. Hoppmann, II, USA

1884. R. H. Scanlan, A note on analysis of beam-columns under arbitrary lateral load and end restraint, J. Franklin Inst. 248, 143–151 (Aug. 1949).

Author discusses a practical solution of title problem by substitution of finite sums for integrals. Ed.

1885. C. C. Miesse, Determination of the buckling load for columns of variable stiffness, J. appl. Mech. 16, 406–410 (Dec. 1949).

Several special problems of buckling of elastic columns with variable cross sections are used to illustrate applications of a powerful iteration method that is described in the book, *Eigenwertprobleme*, by L. Collatz. Method provides upper and lower bounds to approximations of eigenvalues of ordinary linear homogeneous differential equations with homogeneous boundary conditions. Examples that are selected furnish interesting evidence that the procedure frequently converges rapidly.

H. L. Langhaar, USA

1886. W. Barrois and J. Simon-Suisse, Calculation of buckling loads by means of matrices of influence coefficients (in French), Proc. seventh int. Congr. appl. Mech. 4, 52–62 (1948).

Buckling load of a beam under axial forces can be determined by means of matrices whose elements are the influence numbers. Problem is to approximately evaluate an integral whose integrand is the product of two functions g and h, by dividing the interval in n parts, and replacing g and h by constant, linear, or parabolic pieces in each interval. Errors of the method in some examples are between 1 and 4.5 per cent. Th. Pöschl, Germany

1887. K. Roozendaal, Buckling strength of thin profiles (in Dutch with English summaries), Ingenieur 's-Gravenhage 61, 47–49 (July 1949).

Paper gives neither theoretical nor experimental evidence for presented graphs. References are insufficient. Graphs are known by reviewer as those used in German industry during the war (for instance at Junkers). Furthermore, results are incorrect and in part unconservative up to about 20 per cent, as has been shown in experimental investigations by the reviewer (Fokker Reports P 536 and S 54) and others [e.g., F. P. Cozzone and M. A. Melcon, J. aero. Sci. 13, 511–517 (1946)]. H. F. Michielsen, Holland

1888. Elbridge Z. Stowell, Compressive strength of flanges, Nat. adv. Comm. Aero. tech. Note no. 2020, 42 pp. (Jan. 1950).

Author's main purpose is to compute maximum load on a flange with one edge (parallel to load direction) hinged and other edge free. His method is to calculate the elastic strains in the buckled flange, then assume these hold also in plastic range, and calculate the stresses as equal to strains times secant modulus. Thus procedure is essentially, first, to pick a value of twist and from this calculate (using elastic solution derived in paper) the strain ϵ_x; then to determine proper value of E_{sec} corresponding to the strain intensity; and then calculate stress as $E_{sec}\epsilon_x$. This is integrated over the area to give the load. Process is repeated for various starting values of twist, and when a maximum value is found, this is maximum load wanted.

Author claims this seemingly dubious procedure works, and gives various comparisons between calculated quantities and those measured in tests on cruciform sections to prove it. Plots showing strain and rotation distributions show remarkable agreement with experiment. Only comparison involving maximum load is in a plot of σ_{cr}/σ_{cy} against σ_{cr}/σ_{max} (where σ_{cy} = compressive yield stress, σ_{max} = maximum load/area, and σ_{cr} = critical buckling stress). Reviewer would have preferred a direct comparison, say, between calculated and observed curves of end displacement vs. load, to show ability of theory to predict absolute value of maximum load in particular cases.

P. S. Symonds, USA

1889. Ch. Massonnet, Buckling of plates (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 291–300 (Sept. 1948).

A radially compressed circular plate is analyzed, using von Kármán's equation, to show relationship between deflection and load for different degrees of initial eccentricity (assumed parabolic in shape). Effects of membrane stresses in median plane are included. Resulting load-deflection curves are similar to those for eccentric column: initial eccentricity rounds off sharp break in curve and makes it difficult to determine load at which buckling begins. Author shows that initial eccentricity beyond which beginning of buckling becomes difficult to determine is in the order of 3 per cent of plate thickness (for metals). Effect of membrane stresses is to cause marked increase in load-carrying capacity as deflection increases. Large initial eccentricities cause action to approach that of the membrane, in which bending stresses are negligible. Consequently, theoretical buckling load for webs of girders does not necessarily represent usable upper limit. A number of practical observations are made.

Paper, like others dealing with eccentric loading, indicates (to the reviewer) that the word *buckling* has a definite meaning only for concentric loading; when eccentricity is present, problem changes to one of determining a loading limit based on either deflection or bending failure (inelastic action).

F. R. Shanley, USA

1890. C. F. Kollbrunner, Tests on buckling of plates loaded by triangular distributed longitudinal stresses (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 301–307 (Sept. 1948).

In case of plates stressed by unequally distributed longitudinal stresses in elastic zone, the conformity between theory and tests is very good. From author's summary

1891. Mario G. Salvadori, Numerical computation of buckling loads by finite differences, Proc. Amer. Soc. civ. Engrs. 75, 1441–1475 (Dec. 1949).

Concept of central differences of a function, to approximate its derivatives by simple numerical expressions, is introduced. Rectangular, oblique, and triangular coordinates are dealt with and a number of pertinent formulas are presented. It is then shown how a problem involving a linear differential equation of homogeneous type, with homogeneous boundary conditions, may

be reduced to a problem involving solution of a system of simultaneous linear algebraic equations. Successive approximation by increasing the number of intervals seems to be an unsatisfactory approach. A simple method of extrapolation from a limited number of approximations, first suggested by Richardson in 1911, greatly enhances that approach. Errors involved in the procedure are discussed, and tables are given to be used in connection with extrapolation. The procedure is then applied to various cases of buckling to demonstrate technique of its application and to solve a few problems not appearing in the literature. Efficiency of the method of extrapolation, discussed only qualitatively, is obvious from the applications. H. F. Michielsen, Holland

1892. **N. J. Hoff, Dynamic criteria of buckling,** Eng. Struct., N. Y., Academic Press, 121–139 (1949).

Small disturbances near critical load of struts or shells are known to cause buckling. Also during sudden loading of structural systems, inertia effect becomes important. Therefore, author considers the buckling phenomenon as a dynamic phenomenon and gives solution of the differential equation of motion of a rigid bar model which may approximately represent the elastic bar. Bar is hinged at one end and is attached to a spring laterally at other end. A constant axial load acts at end of bar. Motion of this bar is studied with and without fluid and dry friction, and for linear, viscoelastic, and elastoplastic springs. Stable and unstable ranges of axial load are given. Finally, the differential equation for an elastic bar is formulated.

A. Cemal Eringen, USA

1893. **George Wastlund, A method for calculating the safety factor in buckling of fixed-end arches for any variation in moment of inertia** (in Swedish), Inst. Brobyggnad Medd. 3, 3–7 (1948).

Vianello's method is used in conjunction with calculus of differences to solve problem of antimetrical buckling of an arch fixed at both ends, for any variation in moment of inertia.

Points of the center line of the arch are assumed to be displaced radially by an amount w; w thus obeys following differential equation: $w_{ss} + w/\rho^2 = -M_s/EI_s$. Factor of safety against buckling of arch, n, is first determined by neglecting small term w/ρ^2. Then, this approximate value $n_{\text{non corr}}$ may be readily corrected by the aid of the formula

$$n_{\text{corr}} = n_{\text{non corr}}/(1 + EI_s \cos\varphi/H\rho^2).$$

Ch. Massonnet, Belgium

Joints and Joining Methods

(See also Rev. 1952)

1894. **A. J. Francis, Investigations on aluminum alloy riveted joints under static loading,** Eng. Struct., N. Y., Academic Press, 187–216 (1949).

Paper deals with behavior of statically loaded riveted joints of type in which frictional effects are negligible. In first part, joints under axial tensile load are considered. A relaxation method is given for determining theoretically loads carried by rivets in elastic range, and nonelastic behavior is dealt with by means of experiments on aluminum-alloy riveted joints, some of which are described. Both theoretically and experimentally, partition of load is found to be nonuniform at working loads (end rivets carrying more load than inner ones) but to approach uniformity toward failure. It is also shown theoretically that in a joint which fails in rivets, failing load may be considerably less than sum of strengths of individual rivets; a relevant experiment is described.

Second part of paper deals with eccentrically loaded joints. It is found theoretically that in elastic range, loads carried by rivets differ considerably from those given by conventional method. Experiments confirm the theory and also show that, owing to redistribution of load among rivets in nonelastic range, failing load of an eccentrically loaded joint may be appreciably greater than the conventional value, which is obtained by assuming that both under axial load and end couples, rivets are very flexible compared with connected members.

From point of view of the design of joints, the main conclusions of paper are the following: (*a*) To obtain a distribution of load as uniform as possible, it is necessary to keep joints as short as possible by using a small pitch and having few rows of rivets, and to have the two members in joint of equal cross section as far as possible; (*b*) rivets should be at least as ductile as material of members and preferably more so; and (*c*) when fatigue conditions have to be considered, partition of load in region of normal working load—which is more uneven than under failure loading—is very important.

Rules given under (*a*) conflict evidently with generally accepted view on necessity to keep the critical net section of members as large as possible. Reviewer thinks the question is not definitely settled and should be cleared up by further comparative tests. Besides, it should be borne in mind that analysis given here applies to rivets acting effectively in shear. Conclusions drawn therefrom should not be used for riveted connections of steel plates, where frictional resistance plays fundamental role.

Partly from author's summary by Ch. Massonnet, Belgium

1895. **Giulio Ceradini, Resistance to fatigue of lattice girders welded and riveted** (in French), Publ. Int. Assn. Bridge Struct. Engng. Final Rep., 205–213 (1948).

Present paper refers to static and dynamic tests carried out at the L.F.E.M. of Zurich on three test welded girders and two test riveted girders for comparison.

Test results can be considered as very satisfactory and show that it is quite justifiable to consider welded lattice girders, constructed according to fundamental principles of the technique of welded structures, as being the absolute equal of riveted girders, and to allow them to be used equally for structures subject to large dynamic stresses.

Sections of the bars should be simple: in form of a cross or similar. On account of that, results of tests are applicable, when building railway bridges, to those having a span of up to 35 meters (115 ft). From author's summary

1896. **M. Balestra, A method of calculating welded flanges** (in French), Bull. Ass. tech. Marit. Aéro. 46, 663–682 (1947).

Paper deals with analysis of a flange with a tapered collar, welded to the pipe. Instead of usual assumption that flange and collar may be represented by a (thin) plate and shell, approximate three-dimensional stress patterns in flange and collar are assumed. Bending moment transmitted by flange to collar (which is assumed to be free from stresses at weld to the pipe) is determined by condition of equal angle of rotation at joint between flange and collar. It seems that no condition is imposed on radial displacements at this joint. W. T. Koiter, Holland

1897. **D. G. Sopwith, The distribution of load in screw threads,** Proc. Inst. mech. Engrs. 159, no. 45, 373–383 (1948).

Distribution of load along length of a nut is not uniform owing to strains set up in bolt and nut under load. These strains are analyzed and load distribution along thread helix deduced. It is shown that the load may be taken as concentrated at mid-depth of threads. In normal bolt and nut case, maximum intensity of loading occurs at bearing face of nut, and may be from two to four or more times the mean, depending on thread form, propor-

tions of members, and degree of lubrication; maximum intensity is almost independent of length of nut (unless yielding occurs). Possible methods of improving load distribution are discussed—differential pitch, tapered threads, and use of softer material for nut. Load distribution in turnbuckle case is more favorable than in bolt and nut case, and can theoretically be made uniform by suitably boring out inner and shaping outer member; this condition can be approached in practice by use of "tension nuts." Effect of yielding on load distribution is discussed. Experimental confirmation of some results is included.

From author's summary by Thomas J. Dolan, USA

Structures

(See also Revs. 1847, 1869, 1893, 1895, 1943, 2025, 2071)

1898. P. B. Walker, Destructive energy in aircraft pressure cabins, J. roy. aero. Soc. 54, 235–241 (Apr. 1950).

Consideration is given to energy that is suddenly released during a destruction test of an aircraft pressure cabin. Two testing methods are considered as follows: (*a*) Cabin filled with compressed air, and (*b*) cabin filled with water under pressure and immersed in water to compensate for dead-weight effects. Primary purpose of paper is to obtain numerical values for mechanical energy released by the compressed air, by the water, and by the structure. Idealized forms of pressure cabins based on spheres and cylinders are considered. Data are given for ratio of air energy to water energy, air energy to structural energy, and structural energy to water energy. It is concluded that the water testing method is superior from point of view of damage, since the energy released by the compressed air is of the order of magnitude of 10,000 times greater than that of water in a corresponding test.

R. L. Bisplinghoff, USA

1899. Glenn Murphy, The conjugate frame as a tool for evaluating deflections, Proc. seventh int. Congr. appl. Mech. 1, 131–138 (1948).

Author gives an extension of conjugate-beam procedure to frames. He uses a geometrical analysis to show analogy between deflections of original frame and bending moments in conjugate frame. If original frame is fixed at both ends, conjugate frame is the analogous column of Cross' "column analogy." An illustrative problem is given. It is pointed out that conjugate frame may be used as a tool in solution of statically indeterminate structures.

Aris Phillips, USA

1900. Sven G. Bergstrom and Tor-Erik Sundstrom, Comparison of static properties of various types of fixed-end bridge arches and of quantities of concrete required (in Swedish), Inst. Brobyggnad Medd. no. 3, 8–39 (1948).

Comparisons are made of arches in which moment of inertia was varied in four ways, viz., Strassner's, Enyedi's, Kasarnowsky's, and Boussiron's. For the arch proper, the Kasarnowsky arch was most economical in quantity of concrete, and the Boussiron the least economical. It is claimed, however, that substructure required for Boussiron arch is less (due to lower spring moments), particularly in cases where bearing power of ground is comparatively low.

Robert J. Hansen, USA

1901. Ch. Széchy and L. Palotas, The application of prestressing at composite steel plate girder bridges co-operating with the overlying reinforced concrete, Publ. int. Assn. Bridge Struct. Engng. Final Rep., 443–452 (1948).

Paper includes experimental results and theory of prestressing composite steel girder bridges with reinforced concrete decks. A method of prestressing by means of an adjustable central support for simple spans is explained. Lateral distribution of load as influenced by torsional resistance of the longitudinal beams is an important point that is considered. Economic results are indicated.

L. E. Grinter, USA

1902. F. Stussi, Theoretical weight as the basis for selecting a type of bridge (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 475–481 (1948).

Author gives the notion of theoretical weight and extreme span for main girders of bridges. This extreme limit permits indication of limit of economic use of a given type of bridge, so that the weight of the main beams is not exaggerated compared to load borne by the latter (roadway and live load).

In examining present-day structures we find this limit is nearly reached in the cantilever system, whereas in two hinged arches the extreme span is not yet attained. For suspension bridges anchored with high-grade steel cables, the span can still be appreciably increased.

From author's summary

1903. K. Waitzmann, Uniting the floor with the arch of a flat bridge (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 453–438 (1948).

The results of the report have shown: (1) Very good agreement of the theory of Professor Rabut with the results of models tests. (2) Very good agreement of deflections determined on models with deflections recorded during actual tests of bridge. (3) Possibility of solving complex bridge constructions by models with simple laboratory appliances.

From author's summary

1904. R. Chambaud, Experimental investigation on failure in reinforced concrete units under bending (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 671–686 (1948).

Authors carried out experimental study of failure in reinforced concrete parts under bending. Tests included 82 beams of rectangular section and were carried out with great care. They reach the decisive conclusion that classical theory considerably underestimates resistance of compressed concrete in realm of high percentages of stressed steel, and that use of compressed steel is practically never justified.

They propound an elastoplastic theory for calculating concrete units under bending, with a constant safety factor. That theory, which proved to agree with the experiments carried out by the authors, leads to a very important economy in strongly reinforced parts.

From author's summary

1905. Jerzy Mandes, The application of the virtual work equation for calculating wall-beams, Publ. int. Assn. Bridge Struct. Engng. Final Rep., 607–613 (1948).

The principles of calculation are based on application of virtual work equation in form of Ritz-Timoshenko. This method has following advantages: easy choice of stress function, simple equations for stresses, generalizes the results for variable height, includes the problem of buckling, and applies to all kinds of loading by using continuous functions.

From author's summary

1906. K. W. Johansen, Critical notes on the calculation and design of cylindrical shells, Publ. int. Assn. Bridge Struct. Engng. Final Rep., 601–606 (1948).

As the ultimate load must be the starting point for the estimation of the carrying capacity, it is shown how a simple calculation-method based on the rupture stage can be developed. It is illustrated by an example.

From author's summary

1907. **F. Campus, Pre-stressed concrete** (in French), Rev. Metall. 5, 421–436 (Dec. 1949).

Author gives short history of prestressed concrete followed by brief description of basic principles. First part of work will be found useful only by those unfamiliar with the method, but description of tests and of practical realizations which follows is quite interesting. The Hoyer system, in which reinforcement is composed of fully bonded piano wires (*Stahlsaitenbeton*), has been found quite inferior to method where prestress is applied by bearing plates at each end of beam. In concluding, author states that more research is necessary, especially in two different fields: a study of high resistance and high density concrete mixes is of primary importance, as is a better understanding of mechanism of creep or plastic flow in both concrete and cold-rolled reinforcing steel. Robert Quintal, Canada

1908. **J. Naleszkiewicz, On the cooperation of two cantilever spars with a shear-resisting skin,** Arch. Mech. stos. 1, 143–172 (1949).

The author considers a two-spar cantilever wing structure, covered by a thin but shear-resistant skin. The ribs are stiff enough to maintain the shape, and sufficiently close to be considered uniform. The spars are straight but not necessarily parallel. The locus of the shear centers of the cross sections is assumed to be a straight line. The root section is considered to be restrained against warping. The dependent variables are taken to be the symmetrical deflection y and the angle of twist θ; two differential equations connecting them (one for each spar) are derived. Simplification of these equations in some special cases is discussed.

Solutions of the equations are given for a loading consisting of a concentrated load on each spar at any distance c from the root. If the various structural parameters are constant along the spar, it is comparatively easy to solve the equations. If these parameters are variable, any one of the following three methods will give good results: (1) θ is assumed as the sum of a number of functions of the distance from the root, a decreasing correction being applied by successive functions; (2) by the method of Laplace transforms; (3) by finite differences. All three methods are discussed in detail in the paper. Results are presented for the case $c = 0.6L$.

Finally, the problem is solved for the case in which the applied torsional moments are functions of θ (i.e., of the angle of attack). The solution leads to a Volterra integral equation of the first kind, which is solved by an iteration method.

Bruno A. Boley, USA

1909. **Alfred Ducker, On the state of the experimental frost study in highway construction** (in German), Abh. Bodenmech. Grundbau, 169–176, Berlin: Erich Schmidt Verlag (1948).

1910. **J. Dutheil, The design of metallic structures based on plastic deformation** (in French), Ossature Métallique 14, no. 3, 143–162 (Mar. 1949).

Author considers separately plastic readjustment between fibers of same cross section, and readjustment between different cross sections which takes place in hyperstatic systems. Instead of using the maximum moment a bent section can take before failure as criterion for design, author suggests use of a "critical moment" defined as moment producing a permanent set equal to 7.5 per cent of the elastic deformation. A. J. Durelli, USA

1911. **R. V. Southwell, Current trends in structural research,** Eng. Struct., N. Y., Academic Press, 1–8 (1949).

This is an interesting exposition of author's ideas on methods of studying vibrations, shock, buckling, and yielding in structures or structural elements, and energy solutions of statically indeterminate structures. L. H. Donnell, USA

1912. **Kurt Billig, Research work and test production of prestressed concrete units at the Field Test Unit, Ministry of Works, London,** Publ. int. Assn. Bridge Struct. Engng. Final Rep., 367–372 (1948).

The work consisted of two parts: (1) Research on various methods for gripping, fixing, and stretching of compressor wires; bond between the wires and compressed concrete in the prestressed unit, etc. (2) Experimental manufacture and test loading of prestressed concrete units, such as I-joists, floor slabs, and transmission masts. From author's summary

1913. **Erich Friedrich, Recent research on prestressed concrete** (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 359–366 (1948).

With prestressed systems two varying load-stages have to be taken into consideration when laying down a permissible load. First stage is the projection load. This depends mainly on the extent of prestressing. Second stage is the rupture load, which depends upon strength of building material. Report mentions tests which show the connection between the two load stages and measuring processes. From author's summary

1914. **H. Beer and E. Tschech, Elastostatic tests on models of arched dams** (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 627–632 (1948).

Elastostatic investigation on models of arched dams are now proceeding at Technical University at Graz. Report gives a brief view of arrangements for the tests with a modern load apparatus and compares results of deflection measurements on model for a symmetrical wall with calculation made after Ritter method. There is good agreement as to deflection.

From authors' summary

1915. **A. J. S. Pippard and J. E. Duncan, The stresses in an artillery wheel,** Quart. J. Mech. appl. Math. II, 398–411 (Dec. 1949).

Problem under consideration is calculation of stresses in rim and spokes of wheel having spokes with stiffness against bending, and loaded by radial and tangential forces applied at rim. Spokes are replaced by an idealized disk in which neither radial shearing stresses nor circumferential stresses occur (apparently built up by an infinitely large number of spokes), with over-all properties equal to those of actual spokes. This leads to explicit expressions for displacements of rim and actions between disk and rim. Integration of these actions over area corresponding to a particular spoke gives the actions attributed to that spoke. Practical values of parameters for artillery wheels are found to be such that results can be simplified considerably. Experiments were made on wheels with 3, 6, and 12 spokes that show good agreement with calculations. Theory is not compared with the exact one that can be found in Biezeno-Grammel, *Technische Dynamik*.

A. van Wijngaarden, Holland

1916. **Ch. Massonnet, The calculation of plate girders** (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 285–289 (1948).

A plate girder consists of a solid web fitted on its edges with flanges at right angles to web. Author gives the conditions representing the plane state of tension in web at its edges, owing to its connection with flange. Problem of studying tensions in a plate-girder web thus resolves itself into a problem of plane elasticity.

This problem can be solved practically, thanks to a special appliance which will be described in vol. 9 of cited title publication.

From author's summary

1917. Luigi Broglio, A method of "equivalence" applied to the solution of problems of elasticity and of mathematical physics, Proc. seventh int. Congr. appl. Mech. 1, 98–107 (1948).

Author develops an iterative method for solving problems in frameworks, beams, wing structures, etc. He defines a set of data as being "equivalent" to a given set, if it makes the unknowns take on same values as does given set. Hence, superposition of original set of given data (e.g., prescribed load system) together with "equivalent" set—with sign reversed—on given structure, reduces values of the unknowns to zero, and so makes new problem statically determinate and, hence, easily soluble.

Together with the equivalent set—which constitutes a set of self-equilibrating force systems—go a number of simple unit or "elementary" problems, after style of relaxation methods. These are readily soluble, and are used in a simple iterative manner to produce desired solution to the original problem.

Thus, basically, any problem reduces to that of deciding on equivalent problem, and of solving the associated elementary problems.

While claiming broad generality (in title and elsewhere), author illustrates procedure with three examples of framework and beam classes only: (*a*) A plane pin-jointed redundant cantilever structure; (*b*) an arch connected in parallel with a beam by means of a system of rigid pin-ended columns; and (*c*) a continuous beam on nonelastic supports. For last problem the procedure has points of resemblance with moment-distribution process.

F. S. Shaw, Australia

1918. Viktor Haviar, The arch with connected stiffening girder (in German), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 517–532 (Sept. 1948).

Design of steel and reinforced concrete bowstring-girder bridges is frequently simplified by assuming that either the curved rib or the horizontal tie is flexible. Author presents a more rigorous analysis in which the bowstring girder is replaced by an equivalent two-hinged arch of greater and variable flexural rigidity. His assumption that vertical suspenders are hinged at both ends is, however, not always justified as it affects the stresses in these members.

At any nodal point of the girder the vertical deflection is equal to deflection of the rib plus extension of suspender, and from this relation the basic differential equation for the solution of the problem is obtained. Both transverse and axial deformation of members are taken into account in finding the only unknown, namely, thrust of the equivalent arch. Once this has been determined by usual methods, bending moments, shearing, and direct forces in rib, suspenders, and girder follow by the equations given in paper. Method, whose simplicity should appeal to structural engineers, is illustrated by results of influence lines for forces in suspenders of two bridges under construction in Hungary; similar results for a model bowstring show good agreement between calculated and experimental values.

G. G. Meyerhof, England

1919. L. Chitty and Wen-Yuh Wan, Tall building structures under wind load, Proc. seventh int. Congr. appl. Mech. 1, 254–268 (1948).

Paper is a summary of work partially published by L. Chitty (Phil. Mag., London, Oct. 1947). Present paper gives an example of application of the method of analysis to a 20-story wind frame. Method is not developed in detail, but it consists of treating the girders as if they had been replaced by a continuous medium from bottom to top of building. Then the resisting moment per unit of height is determined and transferred into girder moments. Method is shown to agree adequately with slope-deflection for satisfactory application to design. L. E. Grinter, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1860, 1862, 1873, 1938, 1939, 1940, 1948, 1957, 1960, 1964, 1965, 1966)

1920. D. C. Drucker, Some implications of work hardening and ideal plasticity, Quart. appl. Math. 7, 411–418 (Jan. 1950).

Work hardening and ideal plasticity are defined physically and formulated mathematically. Employing these formulations, stress-strain relationships are developed for work-hardenable systems and exemplified by spring-friction models. To assure consistency and uniqueness of solution, the small plastic volume increase which occurs in work hardening must be taken into account, leading of necessity to stress-strain relations of Mises-Prager form. Of functional influence are stress, plastic strain, and history of plastic strain. Hans Winterkorn, USA

1921. J. J. Lynch, E. J. Ripling, and G. Sachs, Effect of various stress histories on the flow and fracture characteristics of the aluminum alloy 24ST, Trans. Amer. Inst. min. metall. Engrs. 175, 435–468 (1948).

Additional evidence on question of fracture strength and retained ductility in tension tests following large prior deformations is presented for 24S-T aluminum alloy. In these tests prestraining was done by extrusion, drawing, compression, and torsion to several degrees of strain in each case. Tension test results for specimens cut from the prestrained material are reported by stress-strain curves, fracture stresses, and fracture strains.

Extrusion and drawing both increased subsequent tensile ductility and fracture stress, increase being roughly proportional to magnitude of prestrain. Extrusion was slightly more effective than drawing. Compression prestrains up to 0.45 in these tests improved fracture characteristics in subsequent tension. Effect of torsional prestrain is qualitatively similar to compression. Strain hardening was identically affected by drawing, extrusion or tension prestrain, apparently because small die angles used here gave a rather uniform and similar condition of flow in all cases.

Experiments are compared with others of same nature that have been reported for other metals, and discussions add more data. The work is an important addition to the literature in this field. Henry A. Lepper, Jr., USA

1922. Alfred M. Freudenthal, The inelastic behavior of engineering materials and structures, New York: John Wiley & Sons, Inc.; London: Chapman & Hall, Ltd., 1950, xvi + 587 pp. Cloth, 5.50 × 8.50 in., $7.50.

Book is divided into three sections: (1) Atomic structure of matter and the mechanism of inelastic deformation, 127 pp.; (2) phenomenological accounts of the response of materials to stress, 241 pp.; and (3) application of mathematical theories of plasticity to technological problems, 143 pp. Author attempts to show how physical and structural considerations can supplement and guide the conventional engineering approach.

An adequate standard could not be maintained by a single author in discussing atomic forces, states of aggregation, statistics of assemblies, mechanism of strain-hardening, fracture, creep, anelasticity, fatigue, impact, rheology of suspensions, classical theory of plasticity, buckling, stresses in forming processes, and mechanical tests. The most successful parts are probably those

on limit design, or on Kelvin and Maxwell solids and associated mechanical models. At the other extreme, the account of the mathematical theory of plasticity and its applications is so compressed as to be unhelpful and misleading. Thus, only three pages are given to the theory of plane strain, with no references to Hencky's basic paper or to recent work. Similarly, rolling and wire drawing are allotted a mere six pages of uncritical mathematics. In view of book's purpose, it is particularly unfortunate that the analysis of stresses in a tube under pressure should be based on what author correctly states to be physically inadmissible stress-strain relations. Assertion that they are the principal tool for solving plastic problems is utterly unfounded, because the relations often lead to unacceptable errors and are in any event intrinsically inapplicable when strains are large.

R. Hill, England

1923. **B. L. Averbach, Plastic deformation,** J. Metals 188, no. 1, 150–153 (Jan. 1950).

The principal strain distribution in a cast tin bar subjected to a 20 per cent rolling was measured by radiographing a lead grid cast into the bar. Principal strains found at various internal positions agreed with those measured on external surfaces in other investigations. A curious fact brought out in present investigation is reversal in sign of lateral principal strain (parallel to roll axis) from tension values at center of bar to compression midway between center and outside.

This novel method of internal strain measurement is similar to that employed by M. Steinberg (Sc.D. thesis, Department of Metallurgy, M.I.T., 1948) and is shown to be a useful experimental tool for measurement of internal deformations in a complex plastic flow state. Louis F. Coffin, USA

1924. **S. B. Batdorf, Theories of plastic buckling,** J. aero. Sci. 16, 405–408 (July 1949).

Author describes stress-strain relations of "deformation" and "flow" types, characterizing these as generalizations of finite and incremental expressions for stress-strain diagram of a simple tension test. He then compares predictions of both types of theory with experimental results. In certain cases, particularly in buckling of plates, better agreement is obtained by use of deformation theory, while in other tests, such as simple tension or compression followed by torsion, flow theory is clearly superior.

Author's new theory of plasticity, which is said to be of neither deformation nor flow type, is claimed to be capable of resolving the contradictions of present theories. In the new theory all plastic deformation is due to slip. Flow occurs first on planes of maximum shear. With further increase of load it then starts on new, slightly inclined planes, when shear stress on them goes above elastic limit. The theory assumes anisotropic strain hardening, since this occurs only on planes on which slip has occurred. The predictions of this theory are said to be in accord with results of experiments of both buckling and compression-torsion types.

Reviewer's comments: The fundamental distinction between stress-strain relations of deformation and flow types is that former are independent of previous loading history, while latter always include such dependence. Any theory must be of one type or the other. Author's theory would appear to be of flow type with complicated anisotropic and nonlinear relations between strain and stress increments. Apparent disagreement with experiment of results for buckling of simply supported plates, based on flow theory, is due, at least in part, to fact that quoted theory considers buckling as true instability with infinitely many possible configurations under the given load. In experiments and deformation-type theory, on the other hand, buckling is considered to occur when the stable bent configuration becomes possible, as in the "Shanley effect" for columns. Thus the results are not strictly comparable. P. S. Symonds, USA

1925. **P. P. Bijlaard, Theory and tests on the plastic stability of plates and shells,** J. aero. Sci. 16, 529–541 (Sept. 1949).

Authoritative paper (with 46 references) summarizes author's numerous contributions to subject, and presents a general theory which appears to be well substantiated by test results. Comparisons with other theories are made and it is shown that some of these are special cases of author's more general theory, which is based on concept that plastic deformations are governed by the amount of elastic shearing energy at proper point. (Paper does not include a discussion of recent theory of Batdorf and Budiansky [NACA T.N. 1871; Rev 1, **1264**] which appears to have a more realistic basis than most previous theories.)

Many different types of loading have been analyzed. In general, it is shown that the plastic buckling stress is increased by any action, during buckling, which increases the shearing stresses. Thus, an edge-supported plate, which must twist in order to buckle, will exhibit relatively large increase in buckling stress; this was also shown by Stowell [NACA T.N. 1556; Rev 1, **825**]. A simple column, however, tends to fail at the stress predicted by the tangent-modulus theory.

Paper should be useful to all investigators in the field of inelastic buckling. F. R. Shanley, USA

1926. **D. C. Drucker, A discussion of the theories of plasticity,** J. aero. Sci. 16, 567–568 (1949).

The author refers particularly to papers by Batdorf and Bijlaard (see preceding reviews 1924 and 1925). Ed.

1927. **R. Hill, Some special problems of indentation and compression in plasticity,** Proc. seventh int. Congr. appl. Mech. 1, 365–377 (1948).

An introductory paper on importance of satisfying velocity as well as stress conditions in plasticity solutions. Mention is made of errors of earlier writers misled by static determinacy of the Saint Venant-Mises stress equations. A solution is given for compression of an infinite wedge of plastic-rigid material by a smooth flat die. In this problems, as in others discussed by Hill and by Lee, the configuration preserves its geometrical similarity as flow proceeds. Later work by author and by Lee makes all these points much clearer. D. C. Drucker, USA

1928. **M. R. Horne, Effect of variable repeated loads in the plastic theory of structures,** Eng. Struct. N. Y., Academic Press, 141–151 (1949).

Problem is to calculate mild steel structures so that even if in some points yield stress is exceeded, safety of carrying capacity is yet intact. Mild steel is said to obey an idealized stress-strain curve, in which strain-hardening effect is not considered. It might be said then that treatise of problem is of minor practical interest. In fact, however, plastic theory and calculation will be necessary to determine residual stresses caused by high load. When residual stresses are known, stresses produced by lower loads can be calculated elastically. Ragnar Nilson, Sweden

1929. **A. Winzer and G. F. Carrier, Discontinuities of stress in plane plastic flow,** J. appl. Mech. 16, 346–348 (Dec. 1949).

Mathematical study following on from W. Prager's postulate that discontinuities of stress can occur in plane "ideally" plastic flow. Justification for stress discontinuities or "shocks" is taken from A. Nadai's sand-heap analogy for torsion of ideally plastic substances. Sharp ridges of sand heap are traces of "shock" planes in this case.

Cases studied here are those when shock separates two regions of nonconstant stresses, and shock intensity varies along trace of shock plane. Method of solution is by parametric substitution for stresses together with power series representation.

Reviewer notes that no experimental evidence is put forward to justify initial assumption that stress discontinuities can occur in plastic flow. It is also noted that equations for all small-deformation theories allow for existence of any finite stress and strain *gradients*. Etched patterns, etc., of plastically yielded zones, published to justify sand-heap analogy, are qualitative and do not preclude existence of steep stress gradients in the physical case in passing from one sand-slope zone to another sand-slope zone. K. H. Swainger, England

1930. Fujio Nakanishi, Yielding of mild steel under tensile load and the relations among the yield points under tension, torsion and bending (in Japanese), Trans. Soc. mech. Engrs. Japan 14, no. 48, part 1, 68–73 (1948).

Author has shown in his former papers that elastic failure or yielding of metals cannot be attributed to maximum value of local stress, but to stability of stress state in whole area of yielding section. He proposed the so-called modified average stress theory, with which he analyzed yieldings under torsion and bending, suggesting that the true yield point under a uniform stress can be found only by pure torsion and bending. In present paper the yield point under a tensile load is conformed experimentally to be about 10% higher than the above true one. Discrepancy is explained by considering that under tensile load a small bending stress is necessarily induced in addition to uniform stress by eccentricity of load center, which is due to plastic deformation during progress of yielding; the state thus becomes yielding under a combined tension and bending stress, and bending stress delays the occurrence of tensile yielding.

Teruyoshi Udoguchi, Japan

1931. J. F. T. Blott and W. B. Bonnor, Yield stresses in lubricating greases, Proc. int. rheolog. Congr. Holland, II: 265–272; III: 71–72 (1948).

Some observations of behavior of grease under conditions of flow are described. Classical assumption of shearing stress flow is adapted to observations, and remarks are made upon observed fact that lubricating greases seem to show two different yield points: one which may be due to influence of adsorption forces, and the second due to some existing rigid structure of soap in the grease. Measurements are made in order to get true confirmation of the flow behavior. The existence of a special type of influence of viscosity and sliding velocity on yield points is found. The investigation deals especially with flow data of calcium greases containing different concentrations of calcium soap.

Ragnar Nilson, Sweden

1932. E. G. Stanford, The creep of metals and alloys, London: Temple Press Limited, 1949, xvi + 162 pp. Cloth, 6 × 8.75 in., $2.10.

In this book of 127 pages and 22 pages of bibliographic references, the reader may find a description of the methods used to measure slow creep of metallic specimens tested under a tensile load at elevated temperatures. This includes descriptions of extensometers, furnace equipment, and of methods for calibration of thermocouples developed in the industrial research laboratories for measurement of long-time creep curves of metals. Various types of such curves are described and a summary is given of metallurgical factors such as grain size, heat treatment, recrystallization, etc., as influencing creep of metals. One short chapter is devoted to presentation of creep results and methods of extrapolating them to service times. One long chapter treats the theoretical physical concepts about mechanisms of creep that have been proposed by the school of physicists (Andrade, R. Becker, Orowan, Kauzmann, Seitz, and others).

The book contains comparatively few details about the valuable theories based on purely mechanical concepts developed by engineers for the treatment of creep problems under simple and combined stress, which are utilized by engineers in their designing of machine parts operating at high temperatures. A. Nadai, USA

1933. R. S. Spencer and G. D. Gilmore, Equation of state for polystyrene, J. appl. Phys. 20, 502–506 (June 1949).

A simplified van der Waals equation of state is proposed to express the relation between pressure, volume, and temperature of polystyrene. This equation is $(P + \pi)(V - W) = RT$, where P is the external pressure, π the internal pressure, V the volume of a basal mole, W volume at absolute zero, T absolute temperature, and R the gas constant. Two mechanisms are responsible for volume changes, i.e., instantaneous and retarded mechanisms. When the temperature is well above the second-order transition temperature, the system is substantially at equilibrium, both mechanisms operate, and thermal expansion measurements reveal an internal pressure of 27,000 psi. At room temperature, in dilute solution or in highly swollen gels most of the volume change is due to instantaneous mechanism alone, in which case an internal pressure of 50,500 psi is expected. This value is in accord with compressibility and sound-velocity measurements at room temperature, and the cohesive energy density computed from observations of the swelling of slightly cross-linked polystyrenes in various solvents. The authors describe apparatus suitable for isothermal compressibility measurements on polystyrene and briefly discuss its use. S. K. Ghaswala, India

1934. Franco Levi, On the generalization of the stress-strain relationship for elastoviscous solids (in Italian), Ric. Sci. 19, 368–373 (May 1949).

Under the assumption that each fiber is formed by the union of elastic and viscoelastic elements uniformly distributed in the cross section, the author generalizes his previously derived law relating stress and strain in a viscoelastic solid [Ric. Scient. 18, p. 586 (1948)] to take into account delayed elastic effects. No application is made. Edward Saibel, USA

1935. A. I. Gubanov, Mechanics of elastoviscoplastic bodies, III: Torsion of a circular cylinder (in Russian), Zh. tekh. Fiz. 19, 773–781 (July 1949).

Paper contains an application of generalized dynamical equations of elastoviscoplastic bodies, which were established by author in first part of the work (Rev 3, **465**). Applying operational method, author finds first the transformed shear stress as a function of angle of twist θ and time t. Then author represents the total applied torque $M(t)$ as sum of two terms corresponding to elastic and plastic zones. In the further calculations author assumes angle of twist θ as a given function of time in three particular cases: (1) θ = const, (2) $\theta = \Omega t$ (Ω = const), (3) $\theta = A \sin \omega t$.

In last part of paper, author examines case of a given torque $M(t)$ and finds that the governing nonlinear differential equation for θ is very complicated. M. T. Huber, Poland

1936. I. I. Gol'denblat, Some general laws of a process of elastoplastic deformation (in Russian), Doklady Akad. Nauk SSSR 68, 1005–1008 (1949).

The author discusses the general mathematical structure of certain nonlinear stress-strain laws for isotropic plastic materials

These laws are all of the "finite" type, and the author does not seem to be aware of the fact that stress-strain laws of this type cannot claim general validity in the plastic range, but apply only under special circumstances when the more adequate "incremental" laws can be integrated to yield finite laws.

W. Prager, USA

1937. **Alfred M. Freudenthal, Work-hardening of metals—a general theory,** J. Franklin Inst. 248, 523–536 (Dec. 1949).

Phenomena which play a role in the process of work hardening are discussed. If the hardness is governed by resilience of the crystal grains alone, it should be inversely proportional to square of average linear dimension of crystallites. On the other hand, according to Taylor's theory of dislocations, and also if it is assumed that increase of hardness is due to storing up of latent energy within intercrystalline boundaries and distorted atomic layers, the hardness would be inversely proportional to these linear dimensions themselves.

A theory is developed that takes account of both possibilities. The form of the general work-hardening function proves to be same for both cases. Experiments are interpreted according to given theory. P. P. Bijlaard, USA

Failure, Mechanics of Solid State

(**See also Rev. 1965**)

1938. **D. C. Drucker, Relation of experiments to mathematical theories of plasticity,** J. appl. Mech. 16, 349–357 (Dec. 1949).

Author discusses several types of mathematical theories of plasticity for work-hardening materials. He first treats problem of choosing the most suitable correlating variable—maximum shearing stress, second-order invariant of stress deviation (equivalent to octahedral shearing stress), third-order invariant of stress deviation—and shows that a function of second- and third-order invariants (J_2, J_3) is needed in general. Next, he points out the weakness of all so-called "deformation-type" theories in their inability to admit existence of a correlation function of both J_2 and J_3. Incremental theories of plasticity with isotropic work hardening are treated next and shown to be applicable under many conditions, but to be unable to cover the Bauschinger effect. Author concludes that a more (though not necessarily completely) general mathematical theory of plasticity must allow for intrinsic anisotropy produced by history of deformation. Such a theory must be of incremental type and must contain terms involving individual total or plastic strains. A series of tests is outlined for comparing merits of maximum shearing stress and octahedral shearing stress as correlating variables. Merit P. White, USA

1939. **J. M. Burgers and G. W. Scott Blair, Report on the principles of rheological nomenclature,** Proc. int. rheolog. Congr. Holland, V: 1–73 (1948).

Design Factors, Meaning of Material Tests

(**See also Revs. 1910, 1927**)

1940. **Joel M. Jacobson, Problems of aircraft life evaluation,** Soc. auto. Engrs. quart. Trans. 3, 616–633 (Oct. 1949).

Author does not report any new information, but reviews thoroughly the field of life evaluation of aircraft parts with many references to fundamental researches conducted by others. Several discussions contribute to make this general review a good approach to the field. A. J. Durelli, USA

1941. **Fernando Vasco Costa, Notions of probability in the study of structures' safety** (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 641–653 (Sept. 1948).

Supposing that probability of ruin of one element of a structure is very small, it is demonstrated that this probability increases proportionally with length of element. As for influence of the section of an element, hypothesis is put forth that resistance of ductile materials increases with cross section, and that resistance of brittle materials decreases when cross section increases, but this must still be confirmed by experiments.

From author's summary

Material Test Techniques

(**See also Revs. 1840, 1864, 1866, 1947, 1958, 1959**)

1942. **Roy W. Hill, George S. Cook, and William E. Moyer, Correlation of accelerated weathering machines,** Amer. Soc. Test. Mat. Bull. no. 164, 32–42 (Feb. 1950).

1943. **J. B. B. Owen, Review of the testing of structures,** Eng. Struct., N. Y., Academic Press, 241–258 (1949).

Techniques for testing of structures to destruction are reviewed principally as they apply to practices in Great Britain. Arrangements of levers, jacks, loading frames, and special supports for pulsating loading machines are described in a general manner. Reviewer believes that the methods of applying brittle resin coatings, and the d-c bridge circuits utilized to measure strains with resistance-wire strain gages are not as advantageous as the current practice in this country; however, an "acoustic" gage is discussed (based on frequency response of a small wire) which may offer some improvement in stability over resistance strain gages. T. J. Dolan, USA

Mechanical Properties of Specific Materials

(**See also Revs. 1921, 1930, 1931, 1932, 1933**)

1944. George V. Smith, Properties of metals at elevated temperatures, New York: McGraw-Hill Book Co., Inc., 1950, x + 401 pp., 238 figs. Cloth, 6.3 × 9.3 in., $7.

Work contains 12 chapters: (1) Introduction, 5 pp.; (2) Plastic deformation of single crystals, 34 pp.; (3) Plastic deformation of polycrystalline material, 55 pp.; (4) Creep of polycrystalline metals, 37 pp.; (5) Fracture of metals, 25 pp.; (6) Theories of flow and fracture, 12 pp.; (7) Tests and test equipment, 51 pp.; (8) Effects of chemical composition on creep, 36 pp.; (9) Effect of melting and deoxidizing practises, heat treatment, and grain size, 39 pp.; (10) Effect of varying temperature and load and of the state of stress, 16 pp.; (11) Changes during service: scaling and microstructural changes, 30 pp.; (12) Design for elevated-temperature service, 10 pp. In addition, there is a list of references containing 488 numbers, an appendix (by R. F. Miller) on highly creep-resisting alloys, and subject and author indexes.

Author gives a thorough review of metallographic aspects of plastic deformation in single crystals and polycrystalline metals, an equally good and detailed report on creep testing and metallurgy of creep, and a short but clear and well-balanced chapter on engineering design under creep conditions. Little space is given to physical interpretation of creep phenomena; much, if not most, of the serious modern work in this field, as well as many classics, is ignored. Thus, Andrade's exponential formula, Becker's 1926 elucidation of contrast between viscous and crystalline types of flow, Mott and Nabarro's work on transient creep, and many other important contributions are not mentioned.

This makes the book more suitable for the physicist whose aim is to collect raw material for his interpretative work than for the metallurgist or engineer who wants to understand the mechanism of creep.

Apart from this limitation, the work is of very high standard and reliability, and reviewer hesitates to point out one or two minor slips. On p. 112 it is suggested that, according to Soderberg's formula, "the strain will be influenced by sojourn at temperature under no stress, an effect impossible to conceive as occurring in metals." However, Soderberg's equations (28) and (29) are obviously meant to give the increase of creep strain at constant stress, and this increase is given correctly as zero if stress is zero. Another minor flaw is the remark, on p. 50, that G. I. Taylor's condition for compatibility of deformation "requires that slip occur on five sets of crystallographically equivalent planes." Correct version is given 12 pages further below.

E. Orowan, England

1945. W. Lethersich, The measurement of the coefficient of internal friction of solid rods by a resonance method, J. appl. Phys. 1, 18–22 (Jan. 1950).

It is usual to assume that longitudinal resonant frequencies of a cylindrical rod are not affected by presence of internal friction. This assumption is sufficiently accurate for metals and certain crystals such as quartz. It is not accurate enough for measurements on plastics, the materials of interest to author. Paper is devoted to introduction of second-order approximations in formulas for computing coefficients of viscosity from vibration data. In the main body of paper it is assumed that, after Stokes, viscous effects arise only from time rates of shear strain. Near the end an additional treatment is given which includes friction arising from time rates of volume changes. It is shown that neglect of second approximation to resonant frequency will introduce errors in internal damping calculations of the order of 12 per cent, whereas its inclusion will reduce them to 1 per cent. It is also pointed out that existence of a volume viscosity can be detected by measurements upon same specimen first with torsion waves, and then with longitudinal. Paper is recommended as a good summary of appropriate formulas for calculating viscous coefficients from vibration experiments which determine width of resonance as conventionally defined.

Horace M. Trent, USA

1946. Paul F. Barrett, Compressive properties of titanium sheet at elevated temperatures, Nat. adv. Comm. Aero. tech. Note no. 2038, 10 pp. (Feb. 1950).

Results are presented of compressive tests of titanium sheets at temperatures ranging from ambient up to 800 F and for exposure times from $^1/_2$ to 2 hr. Material was tested in three conditions: annealed and cold-worked to a 20 per cent reduction, one-half hard, and one-quarter hard. Compressive strengths were found to be maintained reasonably well at elevated temperatures and were comparable to tensile strength. Material was found to be decidedly stronger and more rigid in transverse direction than in longitudinal direction. John E. Goldberg, USA

1947. Joseph Marin, Biaxial tension-tension fatigue strengths of metals, J. appl. Mech. 16, 383–388 (Dec. 1949).

Results of fatigue tests of specimens subjected to stress in two directions are presented. Apparatus used, test specimen, and method of testing are described. Results are given for extruded Alcoa 24S-T material.

Pulsations are obtained by a combination of mechanical and hydraulic methods. Test piece is in form of a hollow cylinder, and oil, given a pulsing motion when passed through a fuel-injection pump, is led through specimen creating alternating internal pressure. Motor driving this oil system also drives a mechanical lever system at same time and this imparts alternating axial loads along the specimen. Machining and preparation of test pieces are given in detail. A calibrated test bar is first used for calibration of gages of testing machine, after which specimen loading is done according to settings obtained. Method of loading and testing is described.

With apparatus used, loadings were obtained in tension-tension quadrant only. Results are presented in form of S-N curves and as tabulation, and these are of several stress ratios investigated. A modified Goodman diagram, as well as a photograph of typical fractures, are also presented.

Existing theories of failure for biaxial strength could not be used to predict failure. Andrew N. Zamboky, USA

1948. J. W. Roderick and I. H. Phillipps, Carrying capacity of simply supported mild steel beams, Eng. Struct., N. Y., Academic Press, 9–48 (1949).

Paper contains comprehensive review of previous work of many investigators on initiation of plastic strain at yield point of mild steel. Of all the theories on bending of mild steel beams, authors prefer that of Robertson and Cook [Proc. roy. Soc. A88, p. 462 (1913)]. They present test data to show agreement between this theory and actual experiments. This preferred theory assumes that stress over cross section of a partially yielded beam increases linearly from zero at neutral axis to upper yield point at some given fiber. Beyond this fiber, stress remains constant at value of lower yield stress. Authors show a great liking for this theory although it assumes that recorded stress-strain diagram of mild steel in neighborhood of yield point represents the actual stress-strain relations.

It is of interest to note that none of the theories discussed in paper takes into account nonuniformity of the strain and, therefore, nonuniformity of the stress distribution along length of a bar in pure bending. Evan A. Davis, USA

1949. A. Krupkowski and S. Kawinski, The phenomenon of anisotropy in annealed polycrystalline metals, J. Inst. Metals 75, 869–880 (July 1949).

Paper deals with anisotropy of rolled sheets and bars, as evidenced by differences in strains measured in thickness and width directions of rectangular tensile specimens. Results are given of tensile tests on annealed specimens of various metals, all specimens being "cut in the direction of rolling." Measurements of thickness and width at various points of specimens, after being pulled to fracture, were used in calculation of certain "coefficients of partial elongation," corresponding to width and thickness directions, respectively. Ratio of these coefficients yields a coefficient of anisotropy for the two directions considered. Some results are given showing variation of this coefficient across thickness of a drawn bar. P. S. Symonds, USA

1950. F. T. Barwell and A. A. Milne, The frictional properties of tungsten carbide, Proc. seventh int. Congr. appl. Mech. 4, 294–310 (1948).

1951. L. J. Klingler, C. C. Chow, and G. Sachs, Flow and fracture characteristics of a die steel at high hardness levels, J. Metals 1, 927–932 (Dec. 1949).

Tension, compression, and bend tests are reported for specimens cut from a 3-in-diam bar of an air-hardening die steel. The specimen axes were taken in different directions from 0 deg to 90 deg with bar axis, and tests were made for material following tempering temperatures from 1000 to 1200 F. Fracture stress

and both tension and bend ductility values were higher in longitudinal directions, while yield stress was not significantly affected by orientation. Anisotropy is attributed to longitudinal carbide stringers developed in forging and rolling rod from an 8-in. ingot.
Henry A. Lepper, Jr., USA

1952. **Friedrich Erdmann-Jesnitzer, The influence of welding-time on spot welding of aluminum alloys** (in German), Z. Metallk. 39, 303–313 (Oct. 1948).

Long welding times of one third, one half and more seconds (20, 30 and more current cycles), according to sheet thickness and stiffness, gave weld nuggets completely free of cracks and pores with use of spherical electrodes and relatively high electrode pressures.

Tendency for pick-up to occur with these relatively long welding times is greater for a grid-controlled welding machine with rectangular envelope of current-time curve than for a cascade-controlled machine with sinusoidal envelope. Longer welding times are likely to cause damage to the glass electron tubes in case of grid-controlled machine, but not in case of cascade-controlled machine. Longer welding time caused no more rejections and is generally recommended for improving quality of spot welds.

Regarding formation of cracks and pores in spot welding of common aluminum-magnesium alloys, an increase of magnesium and manganese is found to be bad and an increase of silicon to be good. Article is followed by a discussion written by Kaufmann and Mohr. They describe a metal electron tube and some of the types of welding current used in Germany.
Wm. C. Johnson, Jr., USA

1953. **Edmund F. Preece, Determination and use of the dynamic modulus of elasticity of concrete,** Highw. Res. Bd. Proc. 28, 233–237 (1948).

A procedure is described by which the elastic modulus of concrete is determined with a sonic analyzer. Since both elastic modulus and flexural strength of concrete increase with age, it is claimed that by performing a sufficient number of observations of both modulus and strength of concrete specimens at different ages, relations can be established, with aid of which the flexural strength at a certain age can be predicted from modulus at an earlier age.
A. M. Freudenthal, USA

1954. **F. Campus, Compact concrete by vibration** (in French), Publ. int. Assn. Bridge Struct. Engng. Final Rep., 345–353 (1948).

1955. **Joseph J. Shideler and Wilbur H. Chamberlin, Early strength of concrete as affected by steam curing temperatures,** J. Amer. Concr. Inst. 21, 273–283 (Dec. 1949).

Testing and results obtained on 990 6 × 12-in. concrete cylinders steam-cured for various periods and at temperatures ranging from 100 to 200 F are discussed. Strength results are given for ages ranging from 6 hours to 28 days, and strengths of companion specimens moist-cured at 70 F are given whenever possible.
From authors' summary

1956. **M. Roš, Influence of the addition of plastocrete in the construction properties of concrete** (in German), Report no. 165 of Eidgenössische Materialprüfungs-und Versuchsanstalt für Industrie, Bauwesen und Gewerbe, Zürich (1948).

Report of tests conducted to determine influence of addition of a commercial dispersion agent, called plastocrete, on commonly measured mechanical properties of concrete. A 1 per cent addition of plastocrete to cement increases the moduli of elasticity and impermeability, diminishes total elastic and plastic deformations, reduces creep, increases frost resistance, while the shrinkage remains unaltered.
A. J. Durelli, USA

1957. **F. Vogt, Effect of shrinkage on the deformation of concrete under sustained loads,** Eng. Struct., N. Y., Academic Press, 217–227 (1949).

It is demonstrated that creep and shrinkage, where the latter is nonuniform over the cross section of a prismatic specimen, are interdependent phenomena. Reason for this is the combination of nonuniform stress distribution plus nonlinear stress-strain relation.

Pickett [J. Amer. Concr. Inst. 13, 333–335 (Feb. 1942)] had previously reached same conclusion from both theoretical and experimental approaches.
C. Martin Duke, USA

1958. **Robert L'Hermite, Recent research in concrete** (in French), Ann. Trav. publics Belg. 102, 481–510 (Oct. 1949).

Author reviews some of the research in concrete carried on at Laboratoires Du Bâtiment et Travaux Publics. Investigations discussed include work on fresh concrete as well as hardened concrete. Most novel work has been done on fresh concrete in an attempt to obtain a fundamental index of workability. A rigidity meter by which one can apply a torsional moment to mass of fresh concrete and measure its angular distortion is described. Reviewer, however, finds several errors; the values of the quality coefficient presented in the table on p. 484 are incorrect, and the equation to obtain unit deformation in the rigidity test does not check dimensionally. Author discusses segregation meter developed by Tournon-Grandvoinnet, which is based on shift of center of gravity of a mass due to the method of placement. Research on hardened concrete covers no original work and involves such problems as size and shape of test specimens, significance of flexural test, sonic testing, effect of curing conditions, and methods of capping specimens for compressive tests.
Frank J. Mehringer, USA

1959. **Robert L'Hermite, The rheology of soft concrete and its vibration** (in French), Rev. Matér. Constr. 405, 179–187 (June 1949).

Author reviews his previous publications on workability of "fresh" concrete, and then presents results of applying his methods to an investigation of vibrated concrete. While the theory does not agree completely with experimental results, it does permit a better understanding of the behavior of vibrated fresh concrete.
Frank J. Mehringer, USA

1960. **J. R. Beatty and J. M. Davies, Time and stress effects in the behavior of rubber at low temperature,** J. appl. Phys. 20, 533–539 (June 1949).

The effect of sustained torsional shear stress on the rate of crystallization of various rubber compounds and mixtures is studied by observing the change with temperature within a range of −80 C and +30 C of the dynamic modulus for different periods of stress application and different stress levels, and by correlating this change with changes of the X-ray diffraction pattern characteristic for crystallization. The test results indicate a considerable accelerating effect of stress on the crystallization rate of rubber at all temperatures within the range of observation. An optimum range which, for unstressed rubber, had been observed at about −25 C, could not be found in the stressed specimens. After removal of the stresses, the dynamic modulus of the specimens tended to return gradually to its original value.

The comparison of measurements of the dynamic modulus with observations of the diffraction pattern indicates the high sensitivity of the modulus to crystallization, since large changes in

modulus could be observed without detectable changes in the diffraction pattern. A. M. Freudenthal, USA

1961. R. Harper, Some recent experimental work on the subjective judgment of the "firmness" of vulcanized rubber, Proc. int. rheolog. Congr. Holland, II: 122–128; III: 43–44 (1948).

The psychophysically defined "firmness" of samples of vulcanized rubber is determined by squeezing a pair of samples between finger and thumb of both hands of the experimenter; the firmness of one of the samples of the pair is always related to the elastic modulus determined by a parallel plate plastometer test. Results are statistically evaluated by probit analysis. Conclusion is reached that differences in the psychophysically determined firmness cannot be accounted for in terms of measurable differences of elastic modulus alone. A. M. Freudenthal, USA

1962. L. R. G. Treloar, The physics of rubber elasticity, New York: Oxford University Press, 1949. 254 pp. Cloth, 5.75 × 8.75 in., $6.

Book is a summary and exposition of some of the theories of rubber elasticity. Initial chapter is largely descriptive of rubber-like substances. Second chapter discusses application of laws of thermodynamics, and shows that elasticity of rubber arises primarily from a change in entropy and secondarily from a change in internal energy. Third and fourth chapters deal with molecular structure; viz., statistical treatment of the single long-chain molecule and development of network theory, respectively.

Chapters V, VI, and VII present a comparison of experimentally obtained stress-strain curves and curves derived from analytical expressions based on the general stored energy function. Results are given for simple elongation, unidirectional compression or uniform two-dimensional extension, and shear.

Photoelastic properties of rubber are discussed in chapter VIII, in which it is pointed out that birefringence is useful not only in determining principal stresses and strains but in obtaining information on flexibility of molecules. Two following chapters treat the subject of crystallization. Chapter XI sets forth results of relaxation or creep tests. Chapter XII treats dynamic properties of rubber, including dynamic modulus and dynamic viscosity, and shows their relation to relaxation effects discussed previously. Chapter XIII concludes the book with development of equations for large strains of several specific examples.

Charles E. Crede, USA

1963. F. T. Barwell and K. W. Pepper, The interlaminar strength of reinforced plastics, Proc. seventh int. Congr. appl. Mech. 4, 278–293 (1948).

Paper presents a comparison of results obtained from several types of test having possible use in determining interlaminar strength of reinforced plastics. Fibrous reinforcement is in form of sheets of paper or fabric, impregnated and bonded together with resin. Tests include shear; transverse tension; compression, both endwise and inclined at 45 deg to laminations; and delamination, a test in which a steel ball is forced into edge of test piece.

Delamination force as a function of shear strength is a straight line, the slope of which is different for shear directions parallel with and transverse to fibers. Correlation of other test results is inconclusive. Tensile test appears to measure strength at weakest plane, instead of strength of material in bulk. Tests in which compressive forces are applied at 45 deg to laminations indicate that laminar planes are capable of withstanding tangential stress at least twice as great as that measured by shearing strength test.

Differences in strength of fabric and paper-reinforced materials are found to result from difference in resin distribution relative to fiber arrangement. When samples are prepared under comparable conditions of cleanliness and impregnation, these differences are virtually eliminated. Charles E. Crede, USA

1964. Donald G. Ivey, B. A. Mrowca, and Eugene Guth, Propagation of ultrasonic bulk waves in high polymers, J. appl. Phys. 20, 486–492 (June 1949).

Paper describes apparatus for experimental determination of ultrasonic velocity in bulk rubbers over a frequency range of 40 kc/sec and from −60 C to 60 C. Curves are shown indicating that wave velocity decreases with increasing temperature and increases slightly with frequency.

Approximate values of dynamic Young's modulus are obtained from the effective modulus under assumption that high-frequency dispersion is due to appearance of a "crystalline" shear elasticity. Results are correlated with low frequency data, and dynamic Young's modulus and loss factor are plotted.

James Woodburn, USA

1965. Alexander Brown, X-ray diffraction studies of the stretching and relaxing of polyethylene, J. appl. Phys. 20, 552–558 (June 1949).

The change, with the extent of mechanical stretching, in the orientation at room temperature and at 96 C of the CH_2-groups in polyethylene, of which between 55 and 73 per cent are present in the form of crystallites, is studied by observation of the change of the X-ray diffraction pattern under different amounts of stretch. At 96 C the observed behavior of the structure is "normal" in that, from the onset of stretching, a preferred orientation develops in which the long-chain axis tends to become parallel to the direction of the stretch; the tendency is weak at low extensions, but increases as the stretching proceeds. At room temperature the behavior is more complex, since within the range of 30 to 200 per cent extension the long-chain axis is inclined to the direction of the stretch at an angle of 64 deg. This is the inclination of the 011-axis in the crystal lattice to the long-chain axis; the 011-axis thus coincides with the direction of the stretch. From 200 to 600 per cent extension, however, this tilt of the crystallites gradually diminishes and finally vanishes near to the fracture point.

No adequate explanation is given for the stretching in the 011-direction. The whole phenomenon, including the necking of the specimen at room temperature, strongly resembles that of texture formation by extensive slip in cold-drawn metal wires; the author discards, however, the possibility of a similar mechanism in polyethylene.

On relaxation of the stretched polyethylene at elevated temperatures, the crystallites gradually become disoriented, though this process shows considerable anisotropy.

A. M. Freudenthal, USA

1966. W. W. Barkas, The swelling of wood under stress, Dept. of Scientific and Industrial Research, Forest Products Res. London, 1949, iii + 103 pp., 40 figs. Cloth, 6 × 9 in., approx. $1.20.

Author carefully develops a theory to predict swelling of wood under stress. In the development he uses the thermodynamic approach. Subject matter can best be discerned from chapter headings: Structure, sorption, and swelling; Anisotropic elasticity; Swelling stresses in elastic gels; Plasticity; and Sorption hysteresis. Author has documented his statements with an extensive bibliography. All sides of debatable items are presented. Author compares theoretical volume hysteresis of beech and sitka spruce with experimental data to measure performance of his theory. Theoretical values are approximately twice the experimental for moisture contents below 15 per cent. Large

discrepancy above this value of moisture content is attributed to accumulation of capillary water, which this theory does not take into account. E. J. Mehringer, USA

Mechanics of Forming and Cutting

(See also Rev. 1949)

1967. **E. K. Henriksen, Residual stresses in machined surfaces,** Trans. Dan. Acad. tech. Sci. no. 7, 94 pp. (1948).

Report presents results of extensive machining tests performed to evaluate residual stresses resulting from various machining techniques. Machine cuts were made on the shaper for low cutting speeds and on the lathe for high cutting speeds. Residual stresses were evaluated from amount of distortion observed in test pieces and expressed as a surface tension force (lb per linear in.). Some attempts to determine stress distribution and depth of strain hardening due to machining are also presented.

The investigation evaluates effect of a number of factors such as depth of cut, width of cut, significant cutter angles and cutting speeds on residual stresses produced.

A brief summary of results indicates: (1) In ductile carbon steel residual stresses are tensile, while in stainless steel and cast iron the stresses are compressive. (2) Stresses are related to work-hardening properties of steel and mechanical action of the cutter, and do not appear to depend very much upon thermal effect of cutting. (3) Residual stresses are concentrated near surface except in the case of austenitic stainless steel where a considerable depth is affected. Ferritic stainless steel shows very low stresses. (4) Magnitude of stresses depends largely upon shape and angles of cutting tool. Wm. Schroeder, USA

Hydraulics; Cavitation; Transport

(See also Revs. 2065, 2114, 2115, 2116)

1968. **J. W. Fox, Onset of turbulent flow in certain arrays of particles,** Proc. phys. Soc. Lond. Sect. B 12, 829–832 (Dec. 1949).

Paper is description of measurements on pressure drop during water flow through three types of packed beds, as a function of volume rate of flow. Beds are (1) perfect hexagonal array of uniform spheres, (2) imperfect array of the spheres, and (3) array of irregularly shaped particles. For very low flow rates the linear (Darcy) variation of pressure drop with volume flow was found, indicating "laminar flow." At some flow rate, slope of log-log plot of these variables begins to increase, reaching values the order of 1.5. This change in slope is said to indicate onset of turbulence. If a Reynolds number is constructed from the particle diameters and mean flow velocity in the empty tube approaching or leaving bed, this "turbulence" appears at a Reynolds number on the order of 20 for the sphere beds and 8 for the irregular particle bed.

Reviewer suggests that (*a*) the word "turbulence" be avoided until flow details in such a system have been investigated; (*b*) if it is eventually discovered that the flow is really turbulent, some more appropriate Reynolds number be found.

Stanley Corrsin, USA

1969. **Herbert Chatley, The distortion of scales in models with loose beds,** Dock Harb. Author. 30, 243–245 (Dec. 1949).

If the Lacey formula $V = 4(2g)^{1/2} R^{2/3} S^{1/3}$ (V is velocity, R hydraulic radius, S slope) is accepted as a universal law, the distortion must equal $y^{1/2}$ (y^{-1} is vertical scale). Author shows some interesting consequences of this distortion law. He states in conclusion that a distortion $y^{1/2}$ may often be adopted in practice, that it may sometimes be increased to nearly $2y^{1/2}$, but that values as high as 100 must be rejected. A. Craya, France

1970. **L. Bonnet and J. Lamoen, Model experiments on the Belgian ports of the North Sea,** Dock Harb. Author. 30: no. 348, 169–174 (Oct. 1949); no. 349, 201–205 (Nov. 1949); no. 350, 237–243 (Dec. 1949).

Extensive hydraulic model tests are described of the Zeebruge and Ostend harbors on the north coast of Belgium. The two harbors differ in that one is protected by a single mole which interferes with the natural currents along the coast line, and the other is built back from the coast and has a narrow inlet which gives little interference with littoral currents. Main problems studied concern improvements to reduce amount of sediment deposited in the harbors and, hence, dredging costs, although some studies were made of wave motion. Since the models had distorted vertical length scales, the velocity and time scales were determined empirically. Authors display considerable ingenuity not only in determining scales, but in interpreting results of the study and in adapting the model to the study of a common but very difficult practical problem. By means of these laboratory studies, authors were able to gain a clear understanding of the sedimentation problems and to recommend comparatively reliable corrective measures. Vito A. Vanoni, USA

1971. **C. N. Davies, The sedimentation and diffusion of small particles,** Proc. roy. Soc. Lond. Ser. A 200, 100–111 (Dec. 1949).

Concentration of particles which settle and diffuse while in suspension is given as a function of time and position for a dilute suspension which is initially homogeneous and contained in a wide vessel. It is assumed that all particles which strike bottom of container adhere, and the walls are so remote diffusion towards them is negligible. Moreover, concentration is taken to be small so that coagulation does not occur and hydrodynamic interference is negligible.

Solutions are obtained for liquid with free surface and also for gas or liquid which fills space between two horizontal planes. The solutions occur as eigenfunction expansions and in integral form. Numerical results are given and graphs are exhibited which show percentage change in concentration as a function of the parameters: distance from free surface, time, and diffusion constant.

C. R. Blomquist, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 2037, 2056, 2080, 2081, 2082, 2115, 2117, 2118, 2127)

1972. **L. M. Milne-Thomson, Theoretical hydrodynamics,** 2nd ed., New York, The Macmillan Company, 1950, xxi + 600 pp. Cloth, $6\frac{1}{4} \times 9\frac{1}{2}$ in., $8.50.

First edition of the book, well known for lucidity of exposition, has been out of print for several years. New edition differs little from the first; chiefly, in addition of a chapter on compressible flow, inclusion of theorems allowing one to calculate potential flow about a circle or sphere from corresponding flow when circle or sphere is absent, and in addition of footnotes referring to more recent literature. Emphasis is upon mathematical theory, little reference being made to experimental work, and in particular, emphasis upon inviscid, incompressible fluid theory to which four fifths of book is devoted. Within these limitations, a wide variety of topics is covered and clearly explained. Reappearance of this book should be welcome to all students of fluid mechanics.

J. V. Wehausen, USA

1973. L. Howarth, Rayleigh's problem for a semi-infinite plate, Proc. Camb. phil. Soc. 46, 127–140 (Jan. 1950).

Paper treats problem of flow induced in a viscous incompressible fluid, initially in a state of rest, by semi-infinite (half-plane) plate which accelerates impulsively from a state of rest to a uniform velocity W parallel to free edge of plate. Solution of the equations in polar coordinates is developed in a series form which converges rapidly for distances from the edge up to $(\nu t)^{1/2}$ which turns out to be the region to which edge effect is limited. For larger distances, the equations are solved by operational methods. These latter solutions are in form of simple integrals which are more convenient for larger distance computations. Solutions are exact. The edge effect in plane of plate, not including plate area, extends twice as far from plate edge as it does for plate region.

Friction forces per unit length of a sufficiently wide strip (i.e., without mutual interference of edge effects) is found for one side of plate to be $\mu W\,[1 + b'(\pi\nu t)^{1/2}]$. When these results are transformed to steady-state case, as Rayleigh did in his treatment of the whole plate problem by replacing t by z/W, then the steady motion solution for a quarter plane moving uniformly and parallel to one edge is obtained. Edge effect is then found to be in interior of a half-parabola region of plate with vertex at corner of plate and axis parallel to motion of plate. In this case, for a sufficiently wide strip, friction forces for one side would be

$$\mu W\,[1 + b\,(W/\pi\nu z)^{1/2}]$$

M. G. Scherberg, USA

1974. O. Holme, A numerical solution of the problem of the lifting surface with continuous distribution of vorticity, Flygtekn. Försöksanst. Medd. no. 32, 20 pp. (1950).

The integral equation for a lifting surface in an inviscid, incompressible flow is solved approximately by introducing the vorticity distribution suggested by H. Blenk [Z. angew. Math. Mech. 5, p. 36 (1925)]. Author points out that Blenk's method of solution did not converge at wing tips. K. Wieghardt [preceding source, 19, p. 257 (1934)] and J. Weissinger [NACA T.M. 1120 (1947)] resolved this difficulty, but Weighardt used only one term in representing spanwise vorticity distribution, and Weissinger only one in chordwise distribution. Author extends Weissinger's results to include an arbitrary number of terms in chordwise distribution, and applies result to rectangular wings, using four (chordwise) terms and tabulating functions that render further calculations for any rectangular wing relatively simple. He states it may be possible to extend method to tapered and swept wings, albeit not without increased difficulties and large computational work. A comparison among various methods, as applied to a rectangular wing of aspect ratio unity, exhibits only small differences for the most part, but Blenk's method appears to yield poor results for center of pressure positions. Reviewer notes that a related attack on the low-aspect ratio lifting-surface problem, which promises to yield useful results without undue labor, has been proposed recently by H. R. Lawrence [see following review].

John W. Miles, USA

1975. H. R. Lawrence, The chordwise lift distribution on low aspect ratio wings and wing body combinations, Cornell Aero. Lab. Rept. AF-673-A-1, Buffalo, N. Y. (May 1950).

(See preceding review.) A systematic procedure is developed for obtaining approximate integral equations from linearized theory of a lifting surface in steady incompressible flow. Procedure leads to a system of classification of various theories currently employed in calculation of lift distribution, and to a new theory for chordwise lift distribution over wings of low-aspect ratio. A certain duality between the high-aspect ratio and low-aspect ratio approximations is exhibited by this classification system. An insight into region of validity of the various approximate theories is obtained.

Following conclusions are established for special case of a rectangular wing, but results are believed to be generally applicable: (1) There does not exist a low-aspect-ratio analog to infinite-aspect-ratio theory. (2) Low-aspect-ratio analog to Prandtl lifting-line theory is Jones low-aspect-ratio theory [NACA T.R. 835 (1946)]. (3) Low-aspect-ratio-analog to Weissinger theory [NACA T.M. 1120 (1947)] is present theory. (4) Weissinger theory and present one are unique among theories under discussion in that they both approach exact lifting surface theory when aspect ratio goes to infinity or zero.

The theory developed for a rectangular wing is extended to more general plan forms. By reinterpreting the integral equation, case of wing-body combinations and more general configurations may be solved. However, regime of validity for this concept must be established by comparison with test data.

An approximate method for solving the integral equation arising in present theory is discussed and applied to calculation of lift-curve slope and aerodynamic center of a series of rectangular wings. Slope of litt curve agrees very closely with Weissinger's theory and test data. Position of aerodynamic center appears to agree with limited test data available.

From author's summary by John W. Miles, USA

1976. O. Björgum, On some three-dimensional solutions of the non-linear hydrodynamical equations, Proc. seventh int. Congr. appl. Mech. 2, Part II, 341–350 (1948).

Author seeks to satisfy Stokes equation of motion of an incompressible fluid moving with vector velocity $\mathbf{v}$ and vorticity ω by writing

$$\mathbf{v} \times \omega = \nabla(p/\rho + \tfrac{1}{2}\mathbf{v}^2),\ \partial\mathbf{v}/\partial t = \nu\nabla^2\mathbf{v}.$$

In this way he arrives at two special exact solutions in the form of Fourier series periodic in x, y, z, and t.

L. M. Milne-Thomson, England

1977. P. N. Brandt-Moller, The growth of circulation around aeroplane wings, Proc. seventh int. Congr. appl. Mech. 2, 140–154 (1948).

An approximation method is given for computation of solution of Wagner's equation for growth of circulation around a wing of infinite span, suddenly put into motion. It is author's intention to extend these calculations to three-dimensional case, using Falkner's method for determination of circulation on loaded surfaces. It is to be remarked that the two-dimensional case was solved analytically by Küssner [Luftfahrtforschung 13, 410–424 (1936); 17, 355–369 (1940)] and by von Kármán and Sears [J. aero. Sci. 5, 379–390 (1938)]. R. Timman, Holland

1978. J. Lamoen, Tides and current velocities in a sea-level canal, Engineering 168, 97–99 (July 1949).

A slightly modified version of Holsters' argument (Rev. 1055) is given and applied to problem of prediction of tidal regime in sea-level Panama Canal connecting the Oceans, without any provision for reducing maximum current velocities. The example is worked out in detail, and Holsters' method is compared, among others, to H. A. Lorentz's method developed in connection with Zuider Zee reclamation. It is pointed out that there are certain special cases in which Holsters' method may fail.

P. Neményi, USA

1979. G. Viguier, Tangential viscosity forces with high velocity gradients, Experientia 5, 397–398 (1949).

Author considers dependence of viscosity coefficient on shear when it is large, in order to explain relation of skin friction to Reynolds number for flow over a flat plate. C. C. Lin, USA

1980. **M. Ray, Development of liquid motion due to an impulse,** Bull. Calcutta Math. Soc. 41, 179–182 (1949).

Particles in the plane $y = 0$ of a viscous fluid of infinite extent are impulsively given a velocity $u(x)$. Resulting motion throughout fluid is then computed by a successive approximation scheme based on the boundary-layer equations, with nonlinear terms omitted in the initial approximation. First two terms of the result are quoted explicitly for $u(x) = e^{-x}$.

G. F. Carrier, USA

1981. **Robert Legendre, An improved solution of Blasius' problem** (in French), C. R. Acad. Sci. Paris 228, 2008–2010 (June 1949).

The author uses a complex stream function ψ, which is connected to the velocity components u and v by the equation

$$u + iv = -2i\nu\,\psi_z,$$

and a total head function φ, which is also a complex function defined by $p/\rho + \frac{1}{2}V^2 = 4\nu\,\varphi_{zz}''$ (where ν is the coefficient of kinematic viscosity, and $p + \frac{1}{2}\rho V^2$ the total head). The functions φ and ψ are functions of the complex variables

$$z = x + iy \text{ iz } = x - iy'.$$

If $F = \varphi + i\psi$, the author reduces the determination of the laminar stream of an incompressible viscous fluid to the solution of equation (1). . . . $8\,F_{z^2}'' + (F_z' - F_z''')^2 = 0$.

In the case of a flow in contact with a flat plate, the solution of (1) is obtained by writing the function F in the form of a sum of a finite number of functions, which are connected by a recurrence relation. The first term of the development corresponds to the solution given by Blasius. The importance of this research, which is undoubtedly remarkable, lies rather in the possibility of applying the method proposed to more complex problems than in the application made by the author (the more so as the corrective functions of Blasius' solution are lacking). However, the paper is so sketchy that the limits and possibilities of the method in question cannot be easily settled. Carlo Ferrari, Italy

Compressible Flow, Gas Dynamics

(See also Revs. 1829, 1972, 2014, 2041, 2058, 2060, 2121)

1982. **S. Tomotika and K. Tamada, Studies on two-dimensional transonic flows of compressible fluid—part 1,** Quart. appl. Math. 7, 381–397 (Jan. 1950).

Paper concerns exact solutions of equations of motion of a hypothetical gas which in neighborhood of sonic velocity has properties similar to those of a real gas. Equations of motion of the real gas in the φ, ψ plane are set up. Relation between density and velocity of the hypothetical gas is then so chosen that certain terms of the equations of motion which are small near sonic velocity are replaced by zero. Properties of resulting hypothetical gas are presented and agree reasonably well with properties of a real gas for velocities between 0.9 and 1.2 times critical velocity.

For the hypothetical gas the equation

$$\left(\frac{\gamma+1}{2}w\right)_{\psi\psi} = \left\{\left(\frac{\gamma+1}{2}w\right)^2\right\}_{\varphi\varphi}$$

is obtained in which w is a variable representing velocity. Some flow patterns which can be derived from two types of solution of this equation are discussed in detail.

One solution discussed is of the form

$$\frac{\gamma+1}{2}w = \Psi_0(\psi) + \Psi_1(\psi)\varphi^2.$$

From this solution a flow pattern representing flow through a first nozzle, an expansion, and a second nozzle is obtained. A figure showing streamlines, constant velocity lines, and Mach lines in physical plane is given. A figure showing the solution in hodograph plane is also presented.

Second discussed solution is of the form

$$\frac{\gamma+1}{2}w = Z(\varphi + \psi^2) + 2\psi^2$$

This solution yields flow patterns for flow through a nozzle with a single contraction. By suitable choice of constants it is possible from this solution to obtain the flow at different transonic velocities through substantially the same nozzle. Figures are shown representing, in both hodograph plane and physical plane, four cases of flow through a certain nozzle. For three of the cases, velocity is subsonic everywhere along nozzle axis, velocity increasing in successive cases. Of these, third case represents the critical flow. The detail behavior of the flow as the critical speed is approached is discussed. The fourth case presented is like the third case on upstream side of the nozzle, but flow on downstream side is supersonic. Hendrik Bruynes, USA

1983. **T. Nonweiler, An empirical treatment of some problems of transonic flow,** Proc. seventh int. Congr. appl. Mech. 2, part II, 437–442 (1948).

A first-order theory is proposed for treatment of plane potential flow in transonic region, where usual linearized theory breaks down. If ϕ is the velocity potential due to a small disturbance in a uniform stream parallel to x-axis, then, to the first order ϕ_x is a function of x only. Hence, writing

$$x' = \int \{(\gamma+1)\phi_x\}^{-1/2}$$

dx, $y - x$, and $y + x'$ are characteristic coordinates and each of the dependent variables ϕ_y and $\phi_x^{2/2}$ may be expressed as the sum of arbitrary functions of these coordinates.

Maurice Holt, England

1984. **Jakob Ackeret, Max Degen, and Nikolaus Rott, Investigation with straight and swept back wings at high subsonic velocities** (in German), Z. angew. Math. Mech. 1, 32–42 (Jan. 1950).

Wind-tunnel drag measurements were made of similar models of different scale to determine blockage corrections at high Mach numbers. Reynolds number of test was kept constant by altering density. An empirical analysis of the data gave results for blockage corrections which differed slightly at the highest Mach numbers tested from the results given by the methods of Thom and Gothert. A. D. Young, England

1985. **A. E. Puckett and H. J. Stewart, The thickness of a shock wave in air,** Quart. appl. Math. 7, 457–463 (Jan. 1950).

Problem of thickness of a shock wave is attacked by authors in a somewhat different way from that of Becker, Thomas, and Bethe and Teller (references given in paper). They assume a Prandtl number of 0.75, as Becker did, but allow C_p, μ, and λ to be functions of absolute temperature, and they assume a thermally perfect gas and linear theories of viscous stress tensor and heat flux vector. Solution is carried through and applied to a particular example in which C_p and μ are constant. Numerical results of 43 L for $M = 1.05$, and 2.5 L for $M = \infty$ are obtained for thickness, where L is a mean free path within shock, and M

is the Mach number. Interpreting this continuum theory in terms of kinetic concepts thus gives number of collisions of a molecule in passing through shock, an interesting point. At time paper was written, authors were no doubt unaware of the experiments of Cowan and Hornig (now available as Technical Report no. 1, "The thickness of a shock front in a gas," Metcalf Research Laboratory, Brown University). Another report giving some indication of effect of Prandtl number on this subject is by Libby, "The structure of a one-dimensional shock wave according to the Navier-Stokes equations" (PIBAL Report no. 152, Polytechnic Institute of Brooklyn). Walker Bleakney, USA

1986. Juergen W. Heberle, George P. Wood, and Paul B. Gooderum, Data on shape and location of detached shock waves on cones and spheres, Nat. adv. Comm. Aero. tech. Note 2000, 69 pp. (Jan. 1950).

Interferograms are presented which show location of detached bow waves for spheres and conical-tip cylinders at Mach numbers from 1.17 to 1.81. Coordinates of waves are tabulated and used for determining empirical expressions for wave shape. Results are compared with theoretical work dealing with detached shock waves. H. Reese Ivey, USA

1987. A. Van Tuyl, On the axially symmetric flow around a new family of half-bodies, Quart. appl. Math. 7, 399–409 (Jan. 1950).

Incompressible flow corresponding to a uniform disk-source placed at right angles to a steady parallel flow from infinity was discussed in a recent paper by A. Weinstein [Quart. appl. Math. 5, 429–444 (1948)]. This flow pattern contains, under certain conditions, a stream surface in form of a half-body whose shape depends on a single parameter. The Blasius-Fuhrman half-body is contained in this family of bodies.

Present paper makes a combined numerical and analytical study of this family of half-bodies and some features of corresponding velocity distributions. Graphs are given by means of which the nose of any given profile can be plotted, and curves are presented which show position and magnitude of maximum velocity on surface as functions of the half-body. All calculations are made, using expressions for stream function and velocity components in terms of elliptic integrals. J. S. Isenberg, USA

1988. Arthur L. Jones and Alberta Alksne, The load distribution due to sideslip on triangular, trapezoidal, and related plan forms in supersonic flow, Nat. adv. Comm. Aero. tech. Note 2007, 37 pp. (Jan. 1950).

Summary is given of load distribution due to sideslip of wings with various plan forms. It is a convenient reference for supersonic aerodynamicists. Chieh-Chien Chang, USA

1989. John W. Miles, A note on supersonic flow in the Trefftz plane, Quart. appl. Math. 7, 470–472 (Jan. 1950).

Velocity potential in a plane infinitely far downstream of a lifting surface is explicitly specified in terms of streamwise perturbation velocity on lifting surface. Paper presents a solution of the linearized equation for supersonic flow by means of Fourier transforms. Lester L. Cronvich, USA

1990. Wallace D. Hayes, Reversed flow theorems in supersonic aerodynamics, Proc. seventh int. Congr. appl. Mech. 2, 412–424 (1948).

By using the linearized equations in cylindrical coordinates for inviscid supersonic flow, fundamental source and lift-element solutions are exhibited, from which author develops his first theorem: if a collection of sources and lift elements are placed in a reversed flow with sign of sources changed and lift vectors unaltered, drag of the system remains unchanged. Proceeding from this result it is shown that thickness drag is unchanged under flow reversal for a variety of geometric objects. Evvard's equations are then used for velocity potential in order to prove that lift-curve slope is unaltered in forward and reversed flow for plan forms with supersonic edges. This result is extended to include plan forms with noninteracting subsonic edges by requring that the Kutta condition be satisfied along subsonic edges. It is stated without proof that plan forms with re-entrant side edges would not have the same lift-curve slope in forward and reversed flow, and hence the reversed-flow theorem for lift cannot be a general one.

Regarding extensions of reversed-flow theorems, reference should be made to the subsequent work of Brown (NACA T.N. 1944) which is based on a method of Munk (Naval Ord. Lab. Memo. 9624). Brown concludes that the reversal theorem for lift-curve slope, as well as thickness drag, applies to an arbitrary plan form regardless of whether subsonic edges interact, or wiether re-entrant side edges exist. On basis of this more recent work it appears that the unproved statement in paper, concerning generality of the reversal theorem for lift-curve slope, may be incorrect. Dean R. Chapman, USA

1991. H. Honl, Two-dimensional wing theory in the supersonic range, Nat. adv. Comm. Aero. tech. Memo. 1238, 34 pp. (June 1949).

Problem of the two-dimensional, oscillating airfoil in a supersonic flow is solved both by the application of Laplace transformation and by a method which is essentially that of Riemann. Since several other solutions are already available (for a complete summary, see Rev 2, **1053**), paper is primarily of historical interest. John W. Miles, USA

1992. G. N. Ward, Supersonic flow past thin wings. I. General theory, Quart. J. Mech. appl. Math. 2, 136–152 (June 1949).

Author deals principally with problem of finding lift of thin plate of arbitrary plan form in supersonic flow. Method of Hadamard is used to overcome the difficulties along Mach cones. Approach is similar to that of Evvard [NACA T.N.1382 (1947); Rev 1, **688**] in which problem reduces to solving an integral equation of Abel type. Closed expressions are obtained for velocity potential for plan forms which are free from interacting subsonic edges. In presence of latter a step-by-step numerical procedure is briefly outlined. (Case in which two subsonic leading edges or two subsonic trailing edges meet at a point is of course excluded.) Lastly, formulas for computing the over-all aerodynamic forces are derived by considering momentum flux through a suitable control surface enclosing the wing.

Hideo Yoshihara, USA

1993. S. Wasserman, Supersonic airfoil theory, J. aero. Sci. 16, p. 760 (Dec. 1949).

Paper offers comments on physical interpretation of derivation of Ackeret's theory. Ed.

1994. Leon Beskin, Supersonic flow past airfoil tips, J. appl. Mech. 16, 329–345 (Dec. 1949).

Paper supplements a previous one by author, entitled "Supersonic characteristics of triangular wings," given at January 1948 meeting of the Institute of the Aeronautical Sciences. It treats, by the Busemann conical flow theory, tip regions of wings of trapezoidal or nearly trapezoidal plan forms, made up of developable surfaces having sections of zero and nonzero thickness.

Flow, synthesized into antisymmetric and symmetric parts, is represented, respectively, by flow over a thick plate at a small angle of attack and flow over a symmetric body at zero angle of attack. Treatment of main subject is preceded by presentation of supersonic flow relations for sweepback and for Busemann conical-flow theory. Markedly distinct solutions are presented for subsonic tips leading, coinciding, and raked to general flow line. This is done separately for antisymmetrical and for symmetrical flow. Author found that, generally, tip base drags are small compared to maximum induced drags, and that it appears an unnecessary refinement to determine such base drags.

Max G. Scherberg, USA

1995. **M. Holt, The numerical method of characteristics for supersonic flows with axial symmetry,** Quart. J. Mech. appl. Math. II, 473–478 (Dec. 1949).

Numerical method of characteristics, applied in its usual form to a segment 12 and the adjacent characteristics triangle 123, introduces at point 3 errors in order ϵ^3 in fields of plane flow, and errors in order ϵ^2 in fields of flow with axial symmetry; ϵ = length of segment 12. Author establishes a second approximation for flow with axial symmetry by means of a simple difference process in which errors of order ϵ^2 may be either calculated or eliminated. By this error correction, avoiding excessive labor, the numerical method of characteristics gives as accurate computed results in fields of flow with axial symmetry as in fields of plane flow. Therefore, this correction will be very helpful in practice.

Robert Sauer, Germany

1996. **R. E. Meyer, Focusing effects in two-dimensional supersonic flow,** Phil. Trans. roy. Soc. Lond. Ser. A 242, no. 844, 153–171 (Dec. 1949).

Steady two-dimensional isentropic supersonic flow is investigated by a geometrical method based on characteristic coordinates. Linear equations developed are essentially those of Nikolskiĭ and Taganov (see NACA T.M. 1213; also REV 2, 348), and by proceeding entirely with geometrical concepts, singularities of the supersonic flow fields are analyzed. A simple expression is obtained for discontinuities of velocity gradient and reflection of such discontinuities at the sonic line. There are a brief presentation and discussion of relation between occurrence of the mathematical limit line and formation of a shock wave.

E. V. Laitone, USA

1997. **Simon Ostrach, Lifting surfaces in supersonic flow. Part I—Two-dimensional theory,** Air Mat. Comm. Dayton Tech. Rep. 102-AC 49/7-100, 59 pp. (Dec. 1949).

Report reviews and summarizes methods currently used to calculate aerodynamic coefficients of airfoil sections in two-dimensional supersonic flow. Consideration is given to elementary linearized theory, to second- and third-order series approximations, and to the so-called "exact" method (sometimes termed the shock-expansion method). An account is also included of theory of infinite sweptback wings. Report should prove useful to anyone engaged in practical calculation of section characteristics at supersonic speeds. It seems unfortunate in this regard, however, that treatment of the higher-order theories gives no consideration to calculation of the pitching moment, since this quantity is the one which is most significantly affected by inclusion of higher-order terms.

Walter G. Vincenti, USA

1998. **E. V. Laitone, Action of viscosity on the supersonic wave drag,** J. aero. Sci. 16, 756–757 (Dec. 1949).

It is shown qualitatively that in two-dimensional supersonic flow over an airfoil, viscosity effects, especially if flow separation takes place, tend to reduce pressure drag, although they have opposite effect in subsonic flow. Reason for this is basically that, whereas in subsonic potential flow compression occurs at rear of an airfoil, in supersonic potential flow expansion occurs over entire rear portion.

Morris Morduchow, USA

1999. **L. Crocco, Air intake and supersonic diffusers** (in Italian), Aerotecnica 29, 131–138 (June 1949).

In subsonic diffusers the best are those in which the diffusion takes place externally between fluid walls. When such a diffuser is used at supersonic speeds, a shock wave develops in front of the entrance and, as the conditions are almost those of normal shock even for the fluid which does not pass through the entrance, the losses are great and increase with the speed. A diffuser can be designed to give normal shock at the entrance and oblique shock at the external edges. This is an improvement, but the shock does not automatically adjust itself. The author inquires whether the internal as well as the external shock fronts can be made oblique. This is answered in the affirmative by an ingenious use of the shock polar. If the vector OA represents the velocity before shock and the vector OP represents the velocity after, it is known that the locus of P is the loop of a folium of Descartes. What is not so well known is that if OP meets the external branch of the folium at Q, then OA represents the velocity after a shock when OQ represents the velocity before. This property can be used to design an intake entrance in which there are two oblique shocks, each turning the stream through the same angle, but in opposite senses so that the final subsonic stream is parallel to the initial stream, and subsonic-diffuser design prevails downstream of the second shock. The author gives a diagram of shock polars adapted to calculate these double oblique shocks. A method of algebraic calculation is also given. The advantages and drawbacks of this form of intake are discussed as well as calculations of efficiency. It is suggested that boundary-layer difficulties can be overcome, at least partially, by the introduction of a slot. Diffusers with three shocks are also suggested, the velocity being still supersonic after two oblique shocks with a final reduction by normal shock well within the entrance. The author remarks that this method would probably increase the global efficiency.

L. M. Milne-Thomson, England

2000. **J. Dörr, Aerodynamic forces on a wing vibrating harmonically in a supersonic flow** (in French), Off. nat. Étud. Rech. aéro. Rep. 37, 29 pp. (1949).

Starting from an elementary solution of the wave equation, an integral equation is derived by superposition for a distribution of dipoles which induces the perturbation field of an oscillating airfoil in two-dimensional sub- and supersonic flow. The results have been known since the time of Küssner's *Allgemeine Tragflächentheorie* [Luftfahrtforschung 17 (1940)]. Using Laplace transforms, this equation is solved explicitly for the supersonic case, and formulas for forces and moments are established.

Applications to a wing with aileron and tab are given. It appears that these cases can be treated with the four transcendental functions introduced by von Borbély (Deutsche Luftfahrtforschung, Forschungsbericht no. 1071) in a much more special case. Extensive tables with numerical results, to be prepared by the title source, are announced.

A. van Heemert, Holland

2001. **Adolf Busemann, A review of analytical methods for the treatment of flows with detached shocks,** Nat. adv. Comm. Aero. tech. Note no. 1858, 23 pp. (Apr. 1949).

Paper reviews analytical methods for treatment of two-dimensional flow with detached shock waves. Emphasis is placed on

supersonic speed range close to Mach number 1.0 where detached shock waves cannot be avoided but where, on the other hand, deviations from potential flow are negligible. Simplifications brought about by two-dimensional potential flow are very desirable with respect to a rigorous combination of the elliptical and hyperbolical flow character in subsonic and supersonic regions. With results obtained in this way, it is easy to see that the connection between body shape and shock-wave shape is far from a simple analytical one which can be represented by a few terms of a power-series development. Between two very sensitive ares of body front is the nose of the body with an unimportant contribution to shock-wave shape. This situation is a significant detail of the transonic problem of detached shock waves, and should be considered in choosing an appropriate method of approach.

Author's summary

2002. H. S. Tsien and Milton Beilock, Heat source in a uniform flow, J. aero. Sci. 16, p. 756 (Dec. 1949).

Pressure, density, and velocity distribution, produced in a compressible inviscid uniform two-dimensional flow by a uniform heat source line normal to plane of flow, are calculated for both subsonic and supersonic cases. Linearization methods are used and heat conduction is neglected. Results are extended to case of a plane source normal to flow plane. It is stated that calculated forces on heat source are zero. Points of interest for two-dimensional source-type heater are that disturbances are distributed in a radial-like manner from heat source in subsonic case, while in supersonic case they are restricted to a narrow band about Mach lines from source. For two-dimensional line type heater, the perturbation velocity vector is normal to line heater in subsonic case, and to Mach wave in supersonic case. Some of the results are presented in terms of a variational function $\delta(y)$ which is either incorrectly or poorly defined. Max G. Sherberg, USA

2003. Lowell M. Bollinger and David T. Williams, Experiments on stability of bunsen-burner flames for turbulent flow, Nat. adv. Comm. Aero. Rep. no. 913, 6 pp. (1948).

Experiments of Lewis, von Elbe and Mentser [J. chem. Phys. 11, p. 75 (1943); 13, p. 89 (1945)] are here extended from laminar into transition and turbulent regions. Blow-off velocities were measured for propane-air Bunsen-burner flames issuing from tubes varying in internal diameter from $^3/_{16}$ in. to $1^1/_8$ in. Fully developed pipe flow was established in each case ahead of burner rim. The mixing phenomenon was simplified by a guard plate mounted perpendicular to, and flush with burner rim. For this nicely defined geometry and flow (except for unknown perturbations of pipe flow by flame) it was shown that blow-off curves are well determined and that fuel-air ratio for blow-off, in range 0.027 to 0.15 is a function only of velocity gradient at wall. For richer flames (fuel-air ratio, 0.15 to 1.0) the scale as well as wall velocity-gradient affects blow-off limit. Data on blow-off and flashback of attached and suspended flames are also discussed.

Bruce L. Hicks, USA

2004. N. Manson, Contribution to the hydrodynamical theory of flame vibration, Proc. seventh int. Congr. appl. Mech 2, I, 187–199 (1948).

This theoretical study of flame and gas movement in closed and open-ended pipes makes allowance for influence of initial disturbance (caused by source of ignition or by burning of first layer ignited); dependence of pressure drop across flame upon partial pressure of active centers in burning gases; and variation of nature of interaction between deflagration wave and disturbances reflected successively from ends of pipe. It is concluded on basis of simplified calculations that, in early stages of flame movement, deflagration vibrations will be excited which may either build up to detonation waves or undergo damping. Resonance vibrations will be expected to occur when flame has transversed about 80 per cent of length of tube. Bruce L. Hicks, USA

2005. Robert Wesley Truitt, Analogy of the special theory of relativity to the study of compressible fluid flow, Engng. Sch. Bull. N. C. State Coll. no. 44, 20 pp. (June 1949).

Author constructs an analogy between the special theory of relativity and the linearized (acoustical) theory of irrotational subsonic flow of a gas. The Prandtl-Glauert rule appears as analog of the Lorentz contraction. C. Truesdell, USA

2006. R. Prim and P. Neményi, On a family of rotational spatial gas flows, Quart. J. Mech. appl. Math. 2, 129–135 (June 1949).

Authors investigate steady flows of an ideal gas in absence of body forces for which the three velocity components in a cylindrical coordinate system based on a plane isometric net depend on only one isometric coordinate. Thus, in a cylindrical coordinate system (ξ, η, ζ), with the square element of arc length

$$ds^2 = g^2(\xi, \eta)(d\xi^2 + d\eta^2) + d\zeta^2,$$

the condition is imposed that log g be a harmonic function of (ξ, η). Reduced velocity fields of following type are considered.

$$W = \xi_1 u(\xi) + \eta_1 V(\xi) + \zeta_1 q(\xi)$$

where W is the ratio of actual velocity V to ultimate velocity magnitude a (constant along any individual stream line), and ξ_1, η_1, ζ_1 are unit vectors in the ξ, η, and ζ directions, respectively.

Four classes of solutions are considered, characterized by following conditions: (*a*) $u = 0$, $\partial g/\partial \eta \neq 0$; (*b*) $u = 0$, $\partial g/\partial \eta = 0$; (*c*) $u \neq 0$, $dq/d\xi = 0$; (*d*) $u \neq 0$, $\partial g/\partial \eta = 0$. After analysis of these solutions, it is concluded that the only plane isometric nets for which it is possible to find steady flows of an ideal gas in absence of body forces, such that the velocity components with respect to cylindrical coordinates based on the net are functions of only one isometric coordinate, are families of logarithmic spirals and their limiting cases. This is a generalization of an earlier result of Tollmien for irrotational flows in case of isometric nets

C. C. Lin, USA

2007. R. Timman, Some remarks on the theory of near-sonic, near-parallel flow and its application to channel flow (in English), Nat. Luchtlab. Amsterdam Rap. no. F. 53, 14 pp. (June 1949).

The paper first derives for the vicinity of the sonic velocity the simplified differential equation and the transonic similarity law. Then it gives two families of particular solutions for the simplified differential equation, and uses one particular solution of the second family in order to represent the flow through a De Laval nozzle. A more detailed discussion of the particular solutions of the second family has recently been carried out by the reviewer (Rev 2, **1161**). Gottfried Guderley, USA

2008. Manfred Schäfer and W. Tollmien, Rotationally symmetric potential flows, Nat. adv. Comm. Aero. tech. Memo. no. 1244, 31 pp. (1949).

This is one of a series of translations from the same German authors, and should be read in conjunction with same source, nos. 1242 and 1243 (Rev 3, **929**). The characteristic equations of rotationally symmetric potential flow in their usual form are extremely difficult to evaluate in practical cases, but the authors have succeeded in so simplifying them that such calculations become much more practicable. The basic forms of the simplified general equations are

$$d\vartheta - \cot\alpha\,(dq/q) \pm \sin\vartheta\,\sin\alpha\,(ds/r) = 0$$

where α is the Mach angle, ϑ the angle of the streamline, q the magnitude of the velocity, r the radial coordinate, and s the distance measured along the characteristic curve. The simplification is obtained primarily by introducing the element of arc ds into the original equations in place of the element of radius dr. The method is applied by way of illustration to the calculation of the flow in a nozzle and in a diffuser. C. W. Smith, USA

2009. **Joseph Peres, On the integration of the equations of the motion of a compressible fluid** (in French), Proc. seventh int. Congr. appl. Mech. 2, 382–387 (1948).

It is shown that hodograph equations for steady irrotational compressible flow, both in terms of stream function and velocity potential and in terms of their Legendre transforms, reduce to an ordinary differential equation of Riccati type. Solution obtained from this equation in general does not represent the motion of a real fluid. However, for certain special values of the coefficients in the equation, which are functions of independent variables, solutions can be made to approximate motion of a real fluid. Relationship between this method and those of von Kármán-Tsien and Chaplygin is shown, and a method of approximation is given where Riccati equation is integrated by quadratures and constants of integration are chosen to approximate real fluid.

Paul M. Naghdi, USA

2010. **T. V. Davies, Unsteady compressible flow in two dimensions,** Proc. roy. Soc. Lond. Ser. A 199, 468–486 (Dec. 1949).

Appropriate differential equations for title problem are developed on basis of linear perturbation theory, which in the present case presumes small departures from a steady basic flow which is compressible but irrotational. In this respect, procedure is similar to that outlined previously by G. F. Carrier and F. D. Carlson [Quart. appl. Math. 4, 1–12 (1946)]. Present treatment, however, employs as independent variables two potential functions which are of the nature of a velocity potential and a stream function.

Specific attention is given to problem of wave propagation when basic flow is circular and is bounded internally by an oscillating circular cylinder. Martin Goland, USA

Turbulence, Boundary Layer, etc.

(See also Rev. 1980)

2011. **H. Blasius, The boundary layers in fluids with little friction,** Nat. adv. Comm. Aero. tech. Memo. 1256, 57 pp. (Feb. 1950).

This is a translation of the classical paper "Grenzschichten in Flüssigkeiten mit kleiner Reibung" [Z. Math. Physik 56, 1 (1908)]. John E. Goldberg, USA

2012. **A. E. Von Doenhoff and L. K. Loftin, Jr., Present status of research on boundary-layer control,** J. aero. Sci. 16, 729–740, 760 (Dec. 1949).

This is a digest of the status of research on boundary-layer control and its application to aeronautical problems. The material is supported by extensive references for specific information. Boundary-layer control by removal of low-energy air and by injecting high-energy air under boundary layer are both considered. Reduction of drag appears to be limited to low Reynolds numbers, and further work is needed to study possible drag reductions at high Reynolds numbers and of effect of irregularities in practical airplane construction. Although maximum lift can be increased some 50 per cent, improvement in low-speed performance (take-off, climb, landing speed) appears unimportant except for high-aspect ratios ($A \geq 15$). There may be an application for thin or swept high-speed wings, but gain over simple leading-edge flaps for straight wings is questionable. Besides subjects indicated, discussion of work on diffusers and inlets of jet airplanes and wind tunnels and of application of boundary-layer control of shock separation at high Mach numbers is included.

Philip Donely, USA

2013. **J. Bass, The foundations of a statistical theory of turbulence** (in French), Proc. seventh int. Congr. appl. Mech. 2, part I, 212–226 (1948).

Author considers statistical theory of turbulence using concepts of mathematical theory of random functions and, in these terms, is able to define very generally, distribution, correlation and spectrum functions for turbulent field. Kinematics of turbulence lead to certain restrictions on functional equation of turbulence, and exact nature of this equation is discussed. Finally, connection with the Navier-Stokes equations is obtained, but instantaneous validity of these equations is not necessary to theory, as it is to classical treatment. A. A. Townsend, England

2014. **J. A. Lewis and G. F. Carrier, Some remarks on the flat plate boundary layer,** Quart. appl. Math. 7, 228–234 (July 1949).

Exact solution of the Oseen equations associated with the flow past a semi-infinite plate is computed, and arguments are presented indicating that this solution is physically not acceptable. A modification of Oseen linearization is introduced such that exact solution of the modified equations completes flow pattern with a reasonable degree of accuracy. Nicholas di Pinto, USA

2015. **K. L. Calder, Eddy diffusion and evaporation in flow over aerodynamically smooth and rough surfaces: a treatment based on laboratory laws of turbulent flow with special reference to conditions in the lower atmosphere,** Quart. J. Mech. appl. Math. 2, 153–176 (June 1949).

A theoretical treatment of the problems of eddy diffusion and evaporation is developed, based on the use of well-established laws of turbulent flow. The semilogarithmic laws for velocity distributions above hydrodynamically smooth and rough surfaces are approximated by power functions, from which the eddy viscosity is obtained as a function of vertical height by differentiation. The molecular viscosity and diffusivity are assumed negligible in comparison with the eddy viscosity up to the surface, and the latter value is taken equal to the eddy diffusivity.

Formulas for the distribution of concentration of the diffusing material downwind from an infinite line source perpendicular to the wind, or from a semi-infinite plane source, are obtained by integrating the diffusion equation in two dimensions. For smooth surfaces the results are identical with those found previously by O. G. Sutton [Proc. roy. Soc. A135, p. 143 (1932); ibid. A146, p. 701 (1934); Quart. J. roy. met. Soc. 73, p. 257 (1947)] and those of W. G. L. Sutton [Proc. roy. Soc. A182, p. 48 (1943)], and are similar to those of Jaeger [Quart. app. Math. 3, 210–217 (1945)]. Calder's treatment does not involve the introduction of Taylor's correlation coefficient or of mixing-length concepts, however, and extends to rough surfaces. The resulting formulas involve the wind velocity at a standard vertical distance above the surface, the exponent of distance in the equation for velocity distribution, the surface shear stress, and a surface roughness parameter.

Experimental velocity distribution data are interpreted to show that flow over earth surfaces covered with closely cut or long grass is "rough." New data on the spread of smoke from a horizontal line source at ground level are in excellent agreement with

the theoretical results, as are also data on the concentration of aniline vapor in air downstream from sprayed strips of grass-covered ground 10 to 30 yd wide in the wind direction.

The assumed velocity distribution applies only to adiabatic conditions in the atmosphere. The treatment may also be applied to the transfer of heat from smooth and rough surfaces provided the temperature differences are not too large.

R. L. Pigford, USA

2016. C. C. Lin, On the law of decay and the spectrum of isotropic turbulence, Proc. seventh int. Congr. appl. Mech. 2, part I, 127–139 (1948).

It is shown that the inverse five-thirds law for spectrum can be derived from general equations for turbulent energy together with some dimensional analysis and the restriction to very high Reynolds numbers. Further, a linear law of decay of turbulence is given (also for high Reynolds numbers) and checked by measurements of G. F. Batchelor and A. A. Townsend [Proc. roy. Soc. Lond. A 190, 534–550 (1947); Rev 1, **323**].

K. Wieghardt, Germany

2017. F. N. Frenkiel, Comparison between theoretical and experimental results on the decay of turbulence, Proc. seventh int. Congr. appl. Mech. 2, 112–126 (1948).

Experimental results seem to agree with many quite different theories, all leading to equations in which turbulence decays exponentially. Much more precise measurements must be made before it will be possible to decide whether certain assumptions as to the nature of turbulence are valid, since all theories investigated apply to homogeneous isotropic turbulence, and since it is not known that such turbulence exists.

In ordinary wind tunnels, more measurements and, in particular, new experimental methods must be available before it will be possible to conclude what theoretical case ordinary wind-tunnel turbulence approaches. Benjamin Miller, USA

2018. Ryozo Fukatsu, The distributions of Reynolds' stress, mixture length, and energy transformed from mean flow to fluctuating flow in the boundary layer (in Japanese), Rep. Inst. Sci. Technol. Tokyo 3, 228–238 (July-Aug. 1949).

With the measured values of mean velocities in boundary layer along a flat plate, author calculates Reynolds stress $-\rho\overline{u'v'}$, mixture length $l = |\overline{u'v'}|^{1/2}/(du/dy)$, and rate of energy transformation from mean flow to fluctuating flow $-\overline{u'v'}du/dy$, using fundamental equations for boundary-layer flow. Experiments are carried out with a flat plate placed in low-turbulence wind tunnel (intensity of turbulence 0.08 per cent, and flat-plate transition Reynolds number 3.7×10^6). Divergent flows are obtained by adjusting the wall opposite to plate. Two kinds of divergent flow are treated: in the one, transition occurs without laminar separation, while in the other, transition occurs after separating flow reattaches to the surface.

Even in region which seems to be laminar before transition, $\overline{u'v'}$ is not zero. Distribution of $-\overline{u'v'}$ suggests existence of an oscillation similar to Tollmien's wave in the inner part of layer. However, additional calculation of $l_1(=u'/(du/dy))$, with values of u' measured with hot-wire anemometer, shows that in outer part of layer, fluctuation is rather similar to Taylor's oscillation.

From beginning to midway of transition region, $-\overline{u'v'}du/dy$ has a maximum at a point fairly distant from surface, and it is to be noted that states of flow near surface (even the separation) contribute very little to this energy transformation. Based on these facts, author states that Tollmien's wave, Taylor's oscillation, or aminar separation may be merely processes which precede transition, and true state of transition develops from a kind of unstable oscillation, something like Tollmien's wave but perhaps more general and three-dimensional, with transformation of energy taking place within relatively wide range with its center about $\frac{1}{4}$ of thickness of layer from surface. After transition is finished, energy transformation is largest in immediate neighborhood of surface. Total transformation energy for accomplishing transition is nearly equal in the two cases considered.

Humio Tamaki, Japan

2019. K. Stewartson, Correlated incompressible and compressible boundary layers, Proc. roy. Soc. Lond. Ser. A 200, no. 1060, 84–100 (Dec. 1949).

Assuming that viscosity is proportional to absolute temperature and that the Prandtl number is one, author is able to relate completely the two-dimensional, compressible, laminar boundary-layer flow over an insulated surface with a certain incompressible flow. Essentially, this method differs from Howarth's [same source, Ser. A 194, 16–42 (1948); Rev 2, **357**] in that the related compressible and incompressible flows do not have same potential-flow velocities. It becomes clear that one effect of compressibility is to emphasize effects of variable potential velocity. New result also shows there are infinitely many "similar" solutions of the compressible-flow equations, contrary to Howarth's conjecture.

The momentum integral is worked out, and also the fundamental equation of the von Kármán-Pohlhausen procedure. Similar solutions are next treated, and then the case of flow near a stagnation point, i.e., potential velocity proportional to coordinate along the surface. Finally, the theory is used to calculate separating boundary layers by von Kármán-Pohlhausen method and Howarth's method [same source, 164, 547–578 (1938)]. If the methods are applied to uniformly retarded potential flow, it is presumed that the separation point should move to leading edge as the Mach number there is increased to infinity. This principle leads author, after some calculation, to conclusion that Howarth's method is preferable for retarded boundary layers.

W. R. Sears, USA

2020. L. G. Whitehead, An integral relationship for boundary layer flow. A new relation for use either simultaneously with, or in replacement of, the von Kármán momentum integral with satisfactory results, Aircraft Engrg. 21, 14–16 (1949).

The two integral relations mentioned in Rev 2, **1170** are examined by using a polynomial of the fourth degree. It is found that the relation obtained by two integrations gives results comparable to the original von Kármán-Pohlhausen method. The use of these two relations jointly gives better results for retarded flow. C. C. Lin, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 1977, 2031, 2037)

2021. Jack D. Brewer and Jacob H. Lichtenstein, Effect of horizontal tail on low-speed static lateral stability characteristics of a model having 45° sweptback wing and tail surfaces, Nat. adv. Comm. Aero. tech. Note 2010, 46 pp. (1950).

Wind-tunnel results are presented illustrating effect of horizontal tail size and location on static lateral stability characteristics of a complete model. Both wing and tail were swept back 45 deg at the quarter chord. Tests were conducted at a Mach number of .13 and a Reynolds number based on mean wing chord of 0.71×10^6. Results indicate that conventional method for predicting effect of an unswept horizontal tail on directional stability does not yield reliable results when applied to swept-tail configurations J. M. Wild, USA

2022. **O. Holme, Comparative wind tunnel tests of a swept-back and a straight wing having equal aspect ratios,** Flygtekn. Försöksanst. Medd. no. 31, 26 pp. (1950).

Comparable results have been published by numerous researchers over the last two to seven years.

John R. Spreiter, USA

2023. **R. H. Miller, Some aspects of the helicopter stability and control problem,** Proc. seventh int. Congr. appl. Mech. 2, part II, 569–583 (1948).

Author has shown how handling qualities of a conventional helicopter can be improved by increasing damping in pitch. By considering characteristics of response of helicopter to control displacements, it is shown that one method is to vary the chordwise and spanwise location of blade balance to obtain a favorable increase in period and damping of pitching oscillations. Paper shows how greatly improved handling qualities may be achieved with only minor design modifications. Theoretical treatment of subject is based upon a dual-rotor helicopter for simplification of mathematical treatment. It is shown, however, that dual-rotor helicopter exhibits same general characteristics as single-rotor helicopter, and indicates that transient response of single-rotor helicopter can be readily obtained from given equations. Consideration is given to fully articulated three-bladed rotor and two-bladed rotor having restrained motion about feathering hinge. It is pointed out that author bases his conclusions on results of calculations of hovering condition. They may or may not be valid for the low-speed forward flight condition that is critical for most helicopter designs. Raymond A. Young, USA

2024. **Kurt H. Hohenemser, Equations for the approximate solution of dynamic problems for stable linear systems,** J. aero. Sci. 16, 723–728 (Dec. 1949).

Dynamic performance of an aircraft is considered (*a*) when elevator is moved gradually from one position to another with a finite rate of change of stick position, and (*b*) when stick is subjected to an impulse-type disturbance. Approximate formulas are developed for numerical or graphical determination of frequency response from a given transient and of a transient from a given frequency response, covering cases of "oblique step" and impulse-type input. Method is an extension of prior studies of other authors related to dynamics of aircraft subjected to "rectangular step"-type stick motion disturbance. Andrew Vazsonyi, USA

2025. **F. J. Plantema and A. van der Neut, Calculation of the deformation of two-spar wings with shear-resistant skin,** Nat. Luchtlab. Amsterdam Rap. 311, S1-S12 (1947).

Paper is abbreviated English translation of Report S. 297 [same source (March 1945)] written in Dutch. Assuming that stresses in wing are known, authors derive equations for determination of deformations of two-spar wings with shear-resistant skin.

Derivations, clearly presented, are based on energy considerations and make use of a modified statement of Castigliano's theorem. Application of variational principles yields five differential equations which, together with appropriate boundary conditions, determine five functions used to specify wing deformations. Paper closes with discussion of numerical method for solving the equations. John R. Spreiter, USA

2026. **W. Sottorf, Systematic model researches on the stability limits of the DVL series of float designs,** Nat. adv. Comm. Aero. tech. Memo. 1254, 49 pp. (Dec. 1949).

To determine trim range in which a seaplane can take off without porpoising, stability tests were made of a plexiglas model, composed of float, wing, and tailplane, which corresponded to a full-size research airplane. Model and full-size stability limits are in good agreement. After all structural parts pertaining to air frame were removed gradually, aerodynamic forces replaced by weight forces, and moment of inertia and position of center of gravity changed, no marked change of limits of stable trim zone was noticeable. The latter, therefore, is for practical purposes affected only by hydrodynamic phenomena. Stability limits of DVL family of floats were determined by systematic investigation independent of any particular seaplane design; thus a seaplane may be designed to give a run free from porpoising provided aerodynamic moments due to center-of-gravity movement, flap deflection, power, and elevator movement for the total flight range do not produce trims in excess of these limits.

Ernest G. Stout, USA

2027. **Lee S. Wasserman, The prediction of dynamic landing loads,** Proc. seventh int. Congr. appl. Mech. 4, 209–220 (1948).

Paper deals with accelerations and loads developed in an airplane due to landing impact. By assuming that landing force increases linearly with time during tire compression and remains constant during oleo compression, author gives a method for computing the vertical load-time history. Load-time curve thus computed is a trapezoid. Drag loads are determined from vertical load using wheel spin-up data. It is assumed that the drag load drops linearly to zero between time t_s of full spin-up and $1.25t_s$.

Calculations of dynamic landing loads are carried out using classical methods for linear systems on assumption that landing-gear loads are independent of vibratory motions of structure. Trapezoidal shape of the load-time curve simplifies calculations, since the response of a linear system to a force which varies linearly with time or to a constant force is relatively easy to compute. Author points out necessity of restricting the number of modes of motion considered in analysis to an absolute minimum in order to keep the time required for numerical solutions within reasonable limits.

In order to check the over-all effect of various simplifying assumptions in analysis, experimental data are compared with calculated results for following cases: (*a*) acceleration at wing tip of a jet airplane with tip tanks; (*b*) acceleration at tail boom of a twin-boom airplane; and (*c*) load in drag link of a four-engine bomber. Agreement between theory and experiment is satisfactory, although in some cases author felt that additional modes should be considered.

Author discusses related work on landing-impact problem and points out where additional work is needed.

Samuel Levy, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 1827, 2000)

2028. **R. Mazet and J. Dörr, Determination by calculation and wind tunnel tests of the critical flutter speeds of a spring mounted, constant profile, rigid model wing** (in French), Off. nat. Étud. Rech. Aéro. Rep. 39, 21 pp. (1950).

Paper reports theoretical and experimental work on influence of position of center of gravity on flutter speeds and frequencies of a simple wing model. Aerodynamically, the wing behaves, with fair approximation, as if of infinite aspect ratio. Mechanically, it is a system with two degrees of freedom—vertical translation and pitching rotation; both degrees of freedom are spring restrained. The mass distribution may be varied to produce differing center of gravity positions. A drop is found in both flutter speed and frequency with aftward movement of center of gravity.

In the theoretical analysis, aerodynamic derivatives are calculated both for an infinitely thin profile as well as for correct profile of the airfoil. These latter calculations are based on paper by G. Couchet (same title source, Rep. 33). Tables of various functions of frequency occurring in Couchet's expressions for the derivatives are given in an appendix. It is found that the influence of profile thickness is extremely small.

Theoretical curves obtained fall slightly below experimental points, even when an appropriate allowance is made for structural damping. Almost exact agreement is obtained, however, if a factor of 0.7 is introduced in all the aerodynamic forces. It is suggested on basis of steady flow tests that a factor of this magnitude is required to account for departures from two-dimensional potential flow. W. S. Hemp, England

2029. Jonathan Winson, The flutter of servo-controlled aircraft, J. aero. Sci. 16, 397–404 (July 1949).

New procedure for solving the flutter problem is given. Wing-aileron system is assumed to be executing neutrally stable flutter at fixed frequency for a fixed forward velocity. Impedance of aileron system (sinusoidal forcing movement divided by angular response) required to maintain the motion is computed. Plots of these impedances for a range of frequencies and velocities are called "flutter boundary charts."

To determine if flutter occurs, actual impedance of aileron system is compared with flutter boundary charts. For the classical flutter problem, impedance of aileron system is simply given in terms of its elastic and damping parameters. When a servo-mechanism links aileron to command station, resulting impedance of aileron depends on servo characteristics and is best obtained experimentally. Stephen H. Crandall, USA

2030. M. J. Turner and James B. Duke, Propeller flutter, J. aero. Sci. 16, 323–336 (June 1949).

A procedure, similar to that used in wing-flutter analysis, is given for the classical torsion-bending flutter of propellers. Generalized coordinates and matrix notation are used. It is found that the coupling between the blades of a propeller is such that it is only necessary to consider the flutter of a single blade with proper boundary conditions. Symmetric and unsymmetric modes are not coupled and hence may be considered separately. An iterative procedure is given for calculating the flexural vibration modes of a nonrotating propeller. A solution is also given for the oscillating air forces of an airfoil oscillating parallel to the flow.

Since an airplane propeller must operate over a wide range of conditions, including the stalled condition, it is the reviewer's opinion that the precise determination of the classical (incompressible, two-dimensional potential flow) flutter speed is of little practical importance. The stall-flutter speed, in general, is only a small fraction of the classical speed, and is therefore the critical flutter parameter. The method outlined in the present paper may, however, be useful when used with the oscillatory stall and transonic airforces (when they become available) to predict the critical flutter condition. Arthur A. Regier, USA

2031. P. Chiarulli, Oscillating wings of finite aspect ratio, Hdqtrs. Air Mat. Comm. Dayton Tech. Rep. 102-AC49/9-100, 23 pp. (Dec. 1949).

Methods of E. Reissner [NACA T.N. 1194 (1947); T. N. 1195 (1947)] and M. A. Biot [Reports 5 and 6, AAF Materiel Center, Wright Field, Ohio (1942)] for determination of aerodynamic loads on an oscillating wing of moderate aspect ratio in incompressible flow are compared by computing lift distribution on elliptic wings of aspect ratio 3 and 6 executing symmetric, torsional oscillations, and by computing flutter damping vs. speed curves for a rectangular wing of aspect ratio 6 with linear torsion and parabolic bending modes. Author concludes that, while Reissner method is more accurate, Biot method is sufficiently close thereto for practical applications in which amount of numerical calculation is an important factor. Author states that it would not seem possible to make a theoretical comparison, but the reviewer notes that, inasmuch as the effects of finite aspect ratio are most important at low frequencies, such a comparison could be made rather directly by expanding the results in powers of reduced frequency and comparing coefficients of first power, since the latter would predominate in determining the damping.

John W. Miles, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1849, 2030)

2032. E. T. Vincent, The theory and design of gas turbines and jet engines, New York, Toronto, London: McGraw-Hill Book Co., Inc., 1950, x + 606 pp. + charts in pocket. Cloth, 5.5 × 9 in., $7.50.

Author's aim is to supply senior and graduate students with information required for a complete understanding of theory and practical design principles relating to rotating fluid machines. Author treats only those theoretical aerodynamic and gas-dynamic problems which concern study of fluid machines, and which can be understood easily by undergraduate students having no specific aerodynamic knowledge. Thermodynamic principles and fundamentals of gas flow, together with more important formulas involved, are recalled and discussed in connection with their application to jet propulsion. Over-all efficiencies of various propulsion devices are compared. Power-plant cycles with and without regeneration are plainly explained and illustrated by means of several examples. The general theory of both jet and power plants employing turbine principle and prediction of their performance characteristics are discussed. A detailed study of theory and design of various units constituting jet and power plants (e.g., centrifugal or axial-flow compressor, turbine, combustion chambers, fuel system, and regenerators) is preceded by an exhaustive description of typical power plants. Fuels and materials are also briefly dealt with. At end of each chapter interesting practical problems are proposed.

Luciano Scholz, Italy

2033. John L. Edward, Design of tail pipes for jet engines including reheat, J. roy. aero. Soc. 54, 217–234 (April 1950).

Paper deals with properties and limitations of a reheat system in tail pipe of a turbojet engine. Due to fact that total pressure in tail pipe is higher than atmospheric pressure, it is possible to increase the momentum of the gas stream by adding heat to it. This can be done by means of burning fuel in tail pipe, only one quarter of the available amount of oxygen being used in the turbojet engine. Author shows that, for economical operation, the reheat system should be combined with a variable nozzle. Specific fuel consumption, although high, has its lowest value if static pressure in tail pipe is as high as possible. This means a low gas velocity and a large cross section, so a compromise has to be made. At fixed turbine conditions and no losses the increase in thrust can be calculated as a function of temperature ratio.

However, some losses are incorporated in the reheat system among which total pressure drop due to heat addition is most important. Author calculates this loss for a cylindrical reheat chamber. For any given Mach number, at entrance of reheat chamber there corresponds a certain temperature ratio, at which the re-

heat-chamber exit is choked. Adding more heat would increase the pressure level in tail pipe, and thus the turbine temperature at fixed engine speed. This may set an upper limit to thrust increase before all available oxygen is used. Author further considers pressure losses due to flame stabilizers and methods for estimating these losses. Attention is paid to combustion efficiency, ignition of the "reheat fuel," and flame stabilization, which is necessary because of high gas velocity. Useful information about test technique, based on author's experience with De Havilland Goblin engine, is given. Paper contains a number of graphs showing results of theory and experiments. J. G. Slotboom, Holland

2034. **H. L. Milner, Aerodynamic balancing of propellers,** J. roy. aero. Soc. 53, 1071–1084 (Dec. 1949).

Paper describes a machine for ascertaining correction angles to be applied in assembling airplane propellers in order to obtain optimum balance in thrust or torque among individual blades. Effective pitch of propeller blade is ascertained by an automatic integrating machine. It forms the sum of the pitch errors, from a previously determined standard pitch, as described by a line between trailing edge and midthickness of blade at center of chord, at specific radial positions. Location of pitch measuring units is determined by a consideration of lift distribution along blade. A discussion of methods used to choose pitch measuring location and of relative importance of thrust and torque unbalance among blades is included. F. E. Reed, USA

2035. **R. Plunkett, A matrix method of calculating propeller-blade moments and deflections,** J. appl. Mech. 16, 361–369 (Dec. 1949).

A method is presented for finding bending moments and deflections of a twisted cantilever beam due to both transverse and axial loads. Actual physical system is approximated by a lumped or discrete system (equivalent to using a discrete approximation to integral equations of actual continuous physical system), equations of which are treated by matrix methods. Solutions of any particular problem may be found by a series of successive numerical integrations. Paper concludes with some remarks on convergence and rate of convergence of the process.

John R. Spreiter, USA

2036. **A. Betz, Influence of gas elasticity on the performance of blade screens** (in German), Ingen.-Arch. 16, no. 3 & 4, 249–254 (1948).

On basis of Bernoulli's equation for compressible fluids, author treats flow conditions of gases in front and in back of blade screens. By physical considerations he presents a synoptical diagram giving relations between pressure and velocity of frictionless fluid in front and back of a vane system. A large range of flow conditions, up to supersonic flow, is given by diagram. Author also investigates how fluid friction can be taken into account when using said diagram. E. Mühlemann, Switzerland

2037. **W. Traupel, On the potential theory of blade cascades,** Sulzer tech. Rev. 2, 12–30 (1948).

A method is described whereby incompressible potential flow over a two-dimensional blade cascade is found for all inlet angles, if blade-surface velocity distribution and outlet angle are known for one inlet angle.

The plane of the blade cascade is mapped conformally on a circle and also on a half plane, and universal functions are computed for general flow fields in these planes. For any specific case, a single iterative procedure yields conformal transformation function. A simple, partially graphical calculation yields expressions for outlet angle and blade-surface velocity distribution for any inlet angle.

Two examples are given for compressor and turbine cascades.

W. G. Cornell, USA

2038. **C. Pfleiderer, Centrifugal pumps for liquids and gases: water pumps, fans, turbo blowers, turbo compressors** (**Die Kreiselpumpen für flussigkeiten und gase**), 3rd revised ed., Berlin, Springer, 1949, XI + 518 pp., 353 figs. Paper, 6 × 9 in. Price DM 51 ($12.24).

The third edition of this standard German work on centrifugal pumps has been revised and broadened in scope by including fans, turboblowers, and turbocompressors. Second edition (1932, 454 pp., available in this country as an Edwards reprint) dealt with liquid pumps exclusively. Now turbomachineries for pumping of compressible fluids have been added. Author justifies this step by pointing to the common physical principles underlying both types of pumping machinery. Treatment of the fluid-dynamics principles has been enlarged and thermodynamic concepts of compressibility phenomena have been added. The flow through the radial impeller is discussed in great detail, combining liquid and gas flow as far as possible. Axial-flow pump is given shorter treatment. Many examples of design calculation and details of construction are included. Difference between pumping of liquid and gas is emphasized in chapters on cavitation and on multistage air compressors. Performance characteristics are discussed in detail, similitudes to a lesser degree.

In spite of this amplification, emphasis remains on the liquid pump, and the beginner who wants to study turbocompressors will find arrangement of the book somewhat to his disadvantage. However, parallel treatment of incompressible and compressible fluid pumping will serve to widen horizon of specialist working in either one of the two fields. O. E. Teichmann, USA

Flow and Flight Test Techniques

(**See also Revs. 1819, 1984**)

2039. **G. B. Schubauer, W. G. Spangenberg, and P. S. Klebanoff, Aerodynamic characteristics of damping screens,** Nat. adv. Comm. Aero. tech. Note 2001, 39 pp. (1950).

Experimental investigation was made of effects of damping screens and performance of screens in oblique flow. Studied characteristics include damping of longitudinal and lateral components of turbulence, effect of screens on scale, conditions for production of turbulence and eddies by screens, and damping of spatial variations of mean speed. Damping theories are reviewed and performance compared with theory. R. C. Binder, USA

2040. **James H. Brown, Design of a schlieren apparatus,** Trend Engng. 2, 15–19, 31 (Apr. 1950).

A brief account describes design features of schlieren system constructed at the Supersonic Laboratory of the University of Washington. Setup in question is a double-pass system with a single parabolic mirror. Second passage of light beam is brought about by a plane mirror mounted behind the transluminated wind-tunnel section, and light is bled off from refocused beam by a half-silvered mirror to form a separate source image on the knife edge. J. F. Weyl, USA

2041. **Rudolf Ladenburg, Application of interferometry to the study of supersonic flow,** Proc. seventh int. Congr. appl. Mech. 2, 388–397 (1948).

Fields of usefulness of Zehnder-Mach interferometers in super-

sonic flows are described. Applications to study of turbulent boundary layers, channel and nozzle flows including their boundary layers, and flow patterns about axially symmetric bodies and two-dimensional objects are shown. Density, temperature, and velocity profiles in boundary layer are given for $M = 1.65$ and 2.3. Results indicate a power law of $^1/_7$ for $M = 1.65$, $Re = 4.1 \times 10^6$, and $^1/_9$ for $M = 2.3$, $Re = 11 \times 10^6$. Lowering test-section pressure to about 5 mm Hg indicated increased boundary-layer thickness and radical changes in density and velocity profiles. H. P. Liepman, USA

2042. George Chertock, A brief survey of the British-type velocity meter, Nav. Res. Lab. Rep. 5-3276, 23–27 (March 1948).

2043. A. Betz, New instruments for flow measurements (in German), Z. angew. Math. Meth. 29, 9–10 (Jan.-Feb. 1949).

The paper contains a very brief description of some new instruments for the measurement of pressures (especially small differences of pressures), and small velocities and their direction.

Albert Schlag, Belgium

2044. W. Wuest, Tubular spring pressure gage (in German), Arch. tech. Messen, no. 155, 65–66 (1948).

Description of different forms of construction of tubular-spring pressure gages: field and precision of reading, system of measuring apparatus and of transmission to reading or recording device.

Albert Schlag, Belgium

2045. Roger D. Schaufele, Note on air-speed indicator compressibility error, J. aero. Sci. 16, 758–759 (Dec. 1949).

A simple computation is made of the inaccuracy of an aircraft air-speed indicator as a function of altitude (sea level to 60,000 ft) and speed (0 to 1000 mph). Discussion is centered around effects of compressibility on Pitot and static pressures.

F. K. Hill, USA

2046. I. Bredt, On a spectroscopic method for determining speeds and temperatures of gases (in French), C. R. Acad. Sci. Paris 228, 1927–1929 (June 1949).

If a gas jet has a velocity V and temperature T, the former can be measured by a spectroscope, using the principle of Doppler-Fizeau, while the latter can be measured with a spectrophotometer, applying the broadening effect of the spectral lines due to the thermal agitation of the molecules.

The Doppler-Fizeau effect produces a displacement of the spectral lines, equal to $\lambda V_r/c$ when a source, emitting light of wave-length λ, moves toward an observer with velocity V_r (c is the velocity of light). (The displacement formula $\Delta\lambda = V_r/c$, quoted in the paper, is incorrect.) If a cold luminous source of light, formed from a suitable reference gas at rest, is placed behind the gas jet so that the incident ray from the source to the spectroscope makes an angle (90 deg $- \alpha$) with the axis of the jet, the relative velocity V_r is equal to $\pm V \sin \alpha$. The wave-length λ_0 of the reference gas can be measured when $V = 0$. The measurement of the displacement $\Delta\lambda(= \lambda_0 V_r/c)$ of the line spectra when the gas jet has a velocity V, enables the velocity to be calculated.

Due to the thermal agitation of the molecules in the gas jet the line spectra will have a "half-width" given by

$$\frac{2b}{\lambda_0} = 7.14 \times 10^{-7} \sqrt{\frac{T}{M}}$$

where T is the absolute temperature of the gas jet (deg K), and M is the molecular weight of the gas.

The "half-width" can be measured by a suitable spectrophotometer and hence the temperature can be found.

If the velocity and temperature are not uniform across the jet, a mean velocity and temperature will be obtained using this spectroscopic method. The lowest velocity that can be measured is approximately 300 fps, but there are no limits to the temperature range.

Finally the author mentions certain precautions which must be noted in applications of this method. G. M. Lilley, England

Thermodynamics

(See also Revs. 1828, 1933, 2004)

2047. Feza Gürsey, Classical statistical mechanics of a rectilinear assembly, Proc. Camb. phil. Soc. 46, 182–194 (Jan. 1950).

Partition function for case of an assembly of a number of identical material points moving on a straight line with interaction between neighboring pairs only is obtained on basis of classical statistics. All thermodynamic properties of such an assembly are known, provided an explicit expression can be found for partition function under an assumed form for the interaction potential.

Author shows that when interaction between neighboring pairs only is taken into account, the phase integral assumes an iterated form which may be transformed by Laplace transformation. Simple asymptotic expressions are then obtained for partition function and density distribution function, from which equation of state follows immediately. In limiting cases of very high and very low temperatures, the partition function obtained for rectilinear assembly of spherical molecules with short-range attractive forces between neighboring molecules only, predicts for assembly the properties of a perfect gas at high temperatures and the behavior of a linear crystal near absolute zero. Due to analytic character of partition function, no critical phenomena are reproduced, but at low temperatures, slope of isothermal curves undergoes extremely sharp changes which may be interpreted as changes of phase.

Mathematical technique employed is not capable of extension to multidimensional assemblies because simplification brought about by the iterated form of phase integral applied to the linear case only. L. M. Grossman, USA

2048. L. Van Hove, Some general properties of a configuration integral of a system of particles with interaction (in French), Physica 15, 951–961 (Dec. 1949).

2049. A. J. F. Siegert, On the approach to statistical equilibrium, Phys. Rev. 76, 1708–1714 (Dec. 1949).

Consideration is given to distribution of a gas as a random process in momentum space as function of time. Probability of changes of distribution in infinitesimal time intervals is assumed to be given by the "Stosszahlansatz." For the Rayleigh model of a gas, probability of a distribution is derived as a function of time. For the Boltzmann gas with microscopic reversibility, it is shown that probability of a distribution approaches the stationary value in the limit of infinite time. From author's summary

2050. D. M. Newitt, M. U. Pai, and J. C. McVeigh, The volume-entropy diagram, Proc. seventh int. Congr. appl. Mech. 3, 19–48 (1948).

Attention is drawn to the thermodynamic diagram whose coordinates are the two extensive properties, volume and entropy. Characteristics of this diagram which differentiate it from other thermodynamic diagrams with coordinates consisting of one in-

tensive and one extensive property are discussed. A small volume-entropy diagram for carbon dioxide is given.

Joseph Kaye, USA

2051. **L. C. Claitor and D. B. Crawford, Thermodynamic properties of oxygen, nitrogen, and air at low temperatures,** Trans. Amer. Soc. mech. Engrs. 71, 885–895 (Nov. 1949).

Increasing importance of low-pressure air-reduction plants requires a recalculation of temperature-entropy and other data at low pressures and temperatures. In this range there may be used as the basic relation an algebraic equation of state, in which reduced virial coefficients are calculated from volumetric, Joule-Thomson effect, and velocity of sound data available in the literature. Results show an internal consistency of 1 per cent or better in range of interest.

Vincent Salmon, USA

2052. **Frederick T. Wall and William R. Krigbaum, Some properties of van der Waals' liquids,** J. chem. Phys. 17, 1274–1278 (Dec. 1949).

Authors test by comparison with physical data certain relationships involving normal boiling temperature, latent heat of vaporization, and coefficients of compressibility and thermal expansion of liquids. These relationships were suggested by manipulations involving a van der Waals equation for liquid volumes, a vapor-pressure equation containing van der Waals constants, and Trouton's rule. Most reliable relation proposed by authors states that product of isobaric coefficient of thermal expansion of a liquid and its molal latent heat of vaporization is equal to 9.4 Btu/lb-mol F at normal boiling temperature. There are notable exceptions to this rule, particularly among inorganic liquids.

H. G. Elrod, Jr., USA

2053. **Glenn C. Williams, Basic studies on flame stabilization,** J. aero. Sci. 16, 714–722 (Dec. 1949).

Action of blunt bodies in stabilizing flames was investigated in steady-flow conditions for a variety of dimensions and temperatures of the bodies and for a range of air-fuel ratios and turbulence conditions of the approach stream. Fuels used were city gas and propane. Both maximum or blow-off velocity and range of air-fuel ratio for a stable flame are discussed as criteria of stability.

Two valuable conclusions are: (1) If span-thickness ratios of bodies are greater than two, blow-off velocity varies about as the 0.45 power of stabilizer size and is independent of shape; and (2) effect of increase in scale or intensity of the approach stream turbulence is to decrease the blow-off velocity.

Other experimental results described are not shown to be generally valid.

Several relevant references not given in paper are: Bollinger and Williams [NACA T.N. 1234 (1947)]; and reports by Williams, Hottel and Scurlock, and by Nicholson and Fields in the "Third symposium on combustion, flame and explosion phenomena" [Williams and Wilkins (1949), pp. 21 and 44, respectively].

Bruce L. Hicks, USA

2054. **R. Mayorcas, Gas temperature measurement above 1500°C,** Proc. seventh int. Congr. appl. Mech. 3, 310–311 (1948).

It appears that no direct method exists of measuring temperatures higher than about 1500 C. It can be shown however that for most heat-transfer problems in industrial furnaces the measurement of the absolute temperature is not the real primary requirement. This may be the direct measurement of heat transfer or of the heat content of the gases involved in a particular process. The former measurements can be made by the heat flowmeter, recently developed in this country, and the latter problem is being approached by the application of continuous gas calorimetry with an instrument called the sensible heat meter.

From author's summary

Heat and Mass Transfer

(See also Rev. 2015)

2055. **E. R. G. Eckert, Introduction to the transfer of heat and mass,** New York, Toronto, London: McGraw-Hill Book Co., Inc., 1950, xiii + 284 pp. Cloth, 6.25 × 9.25 in., $4.

This textbook serves as an introduction to various practical and scientific aspects of transfer of heat and mass. It has the advantage over many similar books that essential formulas required for practical applications are clearly set forth. Considerable attention is given to graphical presentation of results, especially where complete mathematical solutions are not attempted.

After a preliminary chapter on fundamentals of heat transfer, the problem of heat conduction through solids is discussed with special reference to solutions for plates, rods, and fins. For unsteady heat-flow problems the simple graphical method of solution is described in some detail. Use of the electric analogy in heat-conduction experiments is emphasized.

A discussion on the hydrodynamic problem of boundary-layer flow introduces the chapter on heat convection. Emphasis is given to solution of various problems in free and forced convection by use of momentum and analogous equations. Short accounts are written on effect of high speed and low density. Concise chapters on thermal radiation and exchange of mass conclude the book.

Usefulness of this work is greatly extended by detailed solutions of practical problems at the end of each main section. The addition of numerous tables and charts makes it a valuable reference book.

G. M. Lilley, England

2056. **G. F. Carrier and J. A. Lewis, On heat transfer problems in viscous flow,** Quart. appl. Math. 7, 450–457 (Jan. 1950).

Problems treated are reduced to finding temperature distribution governed by conduction from channel walls and by convection for known distribution of velocity and energy dissipation in channel. Characteristic parameter, ϵ, equals (thermal diffusivity/volume flow). The nondimensional temperature variable is proportional to (thermal energy/ϵ × Prandtl number × kinetic energy).

Problems are classified accordingly as ϵ is $\gg 1$, ~ 1, or $\ll 1$, latter giving boundary-layer behavior. Authors' approximate treatment of parabolic equation for $\epsilon \ll 1$ yields interesting boundary-layer solutions by a solution for the interior of flow which does not have proper initial and asymptotic characteristics. For references to earlier investigations, see McAdams, *Heat Transmission* (McGraw-Hill, 1942), p. 187. Bruce L. Hicks, USA

2057. **R. P. Hudson, B. Hunt, and N. Kurti, The use of liquid helium in magnetic cooling experiments,** Proc. phys. Soc. Lond. sect. A 62, 392–94 (June 1949).

The authors describe a simple method of maintaining liquid containers at a constant temperature (in the range of up to 0.5 K) for relatively long periods of time (40 min or longer). They present experimental data on comparative heating-up curves obtained from apparatus of conventional design and their proposed method.

Y. S. Touloukian, USA

2058. **L. Waldmann, On the effect of pressure and temperature on the caloric phenomena accompanying diffusion** (in German), Z. Naturforsch. vol. A4a, 105–117 (May 1949).

The differential equations of diffusion and heat conduction, including thermal diffusion effects, are derived for gases which deviate slightly from ideal behavior. The conduction equation includes, in addition to the thermal diffusion term, a term which varies with the second virial coefficients of the pure gases and of the binary mixtures. Both terms tend to produce temperature differences within an externally insulated mass of gas which is mixing by diffusion. The effect of nonideality is proportional to the pressure. The author shows that at 1 atm the nonideality term is only about 1% of the thermal diffusion term for hydrogen-nitrogen mixtures at 20 C, while the former is 1.6 times larger than the latter for argon-carbon-dioxide mixtures at −53 C. Determinations of the thermal diffusion constant should be made at pressures sufficiently low to avoid the effects of nonideality.

Experiments are described in which various pairs of gases were mixed by diffusion during flow parallel to a screen. Average temperatures measured in each stream by means of resistance thermometers showed that the gas temperature changed as much as 0.4 C due to diffusion. Natural convection caused the effect of pressure to be somewhat different from that calculated, though the thermal diffusion coefficients obtained by extrapolating the temperature changes to zero pressure agreed with values determined independently. A larger effect of pressure on the temperature change of one stream than the other could not be explained by the theory. R. L. Pigford, USA

2059. Mowbray Ritchie, An elementary treatment of thermal diffusion in gaseous and liquid systems, Proc. roy. Soc. Edinburgh 62, part III, no. 32, 305–315 (1948).

Author develops simplified theory of thermal diffusion in a binary gas system based on the postulate that rate of diffusion in region of higher temperature is greater than that in lower temperature region, due to decrease in molecular concentrations in higher-temperature region at constant pressure. Resulting equation is shown to predict results in reasonable agreement with experimental evidence.

Similar expression is developed for thermal diffusion in liquid solutions using diffusion relationship developed for relatively large molecules by absolute reaction rate theory of Glasstone, Laidler and Eyring. W. Kenneth Davis, USA

2060. Shao Wen Yuan, Heat transfer in laminar compressible boundary layer on a porous flat plate with fluid injection, J. aero. Sci. 16, 741–748 (Dec. 1949).

An approximate analysis is made of cooling of porous walls, exposed to flow of hot gases, by means of a coolant injected through walls. Problem is reduced to one amenable to analysis by various simplifying assumptions which include a Prandtl number of unity, a uniform velocity of coolant injection, and a laminar boundary-layer profile expressed as a polynomial of fourth degree. Gerald Nitzberg, USA

2061. F. Salzmann, A method of calculating the temperature distribution in nonuniformly heated tubes, Proc. seventh int. Congr. appl. Mech. 3, 51–63 (1948).

In design of high-temperature heat exchangers, it is important to know tube-wall temperatures as accurately as possible. Method shown in report can be applied to cross-flow heat exchangers where heat flow in direction of tube axis can be neglected, thus making possible a two-dimensional approach. For stationary heat flow, temperature distribution in tube wall is represented by Fourier series. Fourier coefficients are determined by boundary conditions at inner and outer surfaces of tube. Two cases are calculated: tube subjected to external radiation, and tube in a cross-flow gas stream with convective heat transfer. General curves are developed which facilitate calculations. For second case, a simplified iteration method is developed.

Results of the calculations show that temperature differences in the tube wall under nonuniform heat-inflow conditions are very much affected by circumferential equalizing heat flow in tube walls, and by interradiation of inner tube surfaces. These effects have a tendency to level off temperature differences and must therefore be taken into account, especially for high-temperature gas heaters as used in closed-cycle gas-turbine units.

E. Haenni, Switzerland

Acoustics

(See also Rev. 1964)

2062. W. West, The accuracy of measurements by Rayleigh disc, Proc. phys. Soc. Lond. Sect. B 62, 437–444 (July 1949).

König's theoretical formula for torque on a very thin circular disk in a moving stream of fluid contains the numerical coefficient 4/3. Since formula is calculated for an ellipsoid in an infinitely extended, uniformly moving, nonviscous fluid, it is open to corrections arising from departures from these conditions. Of various corrections proposed, some have had effect of increasing, and some of decreasing the numerical coefficient.

Author agrees that corrections for finite thickness of disk, for motion of disk when freely suspended in a stream of alternating motion, and for rotatory inertia of air would each tend to reduce the coefficient somewhat. But he shows that for many of reported experiments these are very small, smaller than probable experimental errors. He disagrees with those who, on basis of their experiments, would increase the coefficient by as much as 10 per cent.

He compares the Rayleigh disk with other methods of calibration of individual microphones, and considers possibility of vibration of diaphragm causing differences in the results.

Gerald Pickett, USA

2063. H. Stenzel, On the computation of the sound field of a circular membrane in a rigid wall (in German), Ann. Phys. Leipzig 4, no. 6, 303–324 (Mar. 1949).

In this purely analytical paper, the sound-pressure field is calculated for membranes whose velocity amplitude may be expressed as a power series in $1 - (\rho/a)^2$, where ρ is the radius of an annular element of the membrane, and a is the outer radius. The sound pressures on the axis, and at large distances r, are given by fairly simple expressions. These are used as leading terms for developing various series expansions for the sound pressure in three spatial regions determined by r, wave-length λ and azimuthal angle γ. For $a < r < \pi a^2/\lambda$ and $\gamma \leq 45$ deg, the preferred expansion utilizes the axial sound pressure, with power of $\sin\gamma$. When on the other hand, for the same distances, $\gamma > 45$ deg, the expansion is in terms of the distant sound pressure and powers of $\cos\gamma$. For $\pi a^2/\lambda \leq r < \infty$ and γ unrestricted, expansion is the the usual one in terms of Legendre functions. Tabulated values are presented by the author for the Rayleigh (rigid) piston for $ka = 10$ and kr from 20 to 100. In general, convergence is the worst near 45 deg, where ten or more terms are required. The paper concludes with an application to a pair of radiating coplanar elementary surfaces. Vincent Salmon, USA

Ballistics, Detonics (Explosions)

2064. A. Signorini, On some problems of rigid dynamics, Proc. seventh int. Congr. appl. Mech. 4, 226–236 (1948).

Problems considered are the following: (1) Unrestricted problem of exterior ballistics. Author summarizes his results pub-

lished elsewhere [Mem. Acc. Lincei 1, 1–41 (1946)] on reduction of the order of equations to lowest possible order. (2) Extension of Culmann's ellipse to field of rigid dynamics [cf. Sopra alcune questioni di geometria delle masse, cinematica delle masse e stereodinamica, Ann. Mat. pura appl., 1–11 (1945). (3) Precessional rotations of an unsymmetric heavy top [cf. G. Grioli, Precessioni regolari di un solido pesante asimetrico, Rend. Acc. Lincei (Apr. 1948)]. D. C. Lewis, USA

2065. **M. O. Wolfe, The measurement of fluctuating fluid pressures,** Aircr. Engng. 21, 368–377 (Dec. 1949).

Excellent summary of the more important developments in methods of measuring fluctuating fluid pressures. Fundamental requirements of pressure pickups are discussed at length. Various types of pickups are described, and their limitations and advantages are given. Thus, sufficient information is given to judge suitability of a particular pickup and its associated electronic recorder for a specific application. Types of pressure pickups discussed are: (1) resistance, (2) electromagnetic, (3) electrostatic, (4) photoelectric, and (5) mechanical devices such as the "Farnboro" indicator. Some design information is given for condenser-type pickups, and methods of dynamic calibration are suggested. P. Rosenberg, USA

2066. **P. Carriere, The method of characteristics applied to the problems of internal ballistics,** Proc. seventh int. Congr. appl. Mech. 3, 139–153 (1948).

Paper contains a one-dimensional solution for no viscosity or heat transfer. Method of characteristics is used in variables, log (density), entropy, and usual space time co-ordinates. Two specific examples are evaluated, powder rocket and gun, but graphical results given are not of practical use.

R. C. Knight, England

Soil Mechanics, Seepage

(See also Revs. 1968, 2114)

2067. **C. C. Miller, A. B. Dyes, and C. A. Hutchinson, Jr., The estimation of permeability and reservoir pressure from bottom hole pressure build-up characteristics,** J. petrol. Tech. 2, 91–104 (April 1950).

From theoretical considerations equations are developed relating pressure build-up at bottom of an oil well after shut-in to properties of formation around well. By comparing measured build-up pressures versus logarithm of time since shut-in with theoretical values, estimates are made of reservoir pressure, average permeability of formation, and nonuniformity in permeability with distance from well. Problem was simplified by assuming steady-state production prior to shut-in. Consequently, it was desirable that the well be producing long enough to be practically stabilized prior to testing.

The early part of the build-up curve gives information as to whether permeability in immediate vicinity of well is greater or less than that farther from well. Damage due to drilling operation may reduce permeability, and special treatment may increase it. For detailed study of this variation of permeability, authors recommend that theoretical values be obtained by means of an electrical analog, since nonuniformity in permeability introduces considerable complexity into theoretical equations.

Curves are shown representing data from seven of 80 wells studied. Examples are given illustrating method of calculation and steps to be taken to arrive at desired estimations. Criteria are given to indicate how soon a test may be started, and how long a test shall be run to obtain sufficient data.

The fact that the extent of damage to permeability due to drilling or effectiveness of special treatments to increase permeability can be readily determined by method of analysis presented, may be of considerable practical importance.

Gerald Pickett, USA

2068. **W. J. Leas, L. H. Jenks, and Charles D. Russell, Relative permeability to gas,** J. petrol. Tech. 2, 65–72 (Mar. 1950).

A detailed description is given of an experimental method for determining relative permeability of porous media to gas. Results are presented for natural and synthetic cores. Experimental data for two- and three-phase systems are compared. Reproducibility of the equilibrium gas saturation is demonstrated, and effects due to variation in rate of gas flow are discussed. Attempt is made to correlate total permeability and rock textural factors with relative permeability to gas. Capillary pressure curves obtained simultaneously with relative permeability curves are shown to be equivalent to those obtained in conventional displacement cells. From author's summary by R. E. Fadum

2069. **E. W. Vaughan, Field bearing tests verify laboratory findings,** Civ. Engng. 19, no. 10, 24–27 (Oct. 1949).

The author compares the results of tests performed on weathered schist at the site of Agua Fria Dam near Caracas, Venezuela. Almost perfect agreement was found between field-test results obtained from two different test pits, and good agreement was found between field tests and laboratory shear tests. Ed.

2070. **Johann Ohde, Preloading and prestressing of foundation soil and their effect on settlement, stability and sliding resistance** (in German), Bautechnik 26: 129–134 (May 1949); 163–167 (June 1949).

The author points out the importance of preloading and prestressing with regard to the settlement and resistance of subsoils. He first discusses the effect of preloading and prestressing on the well-known pressure-voids ratio curve, and discriminates between the concepts of preloading and of prestressing, as the first one is the maximum load which ever acted, while the latter represents a part of the present pressure. He gives several examples showing that the difference in stress distribution and settlement depends upon the rate of loading and on the nature of subsoil. The behavior of pore water in cohesive soils plays a preponderant role which is difficult to express by exact formulas, and only some approximations are given. Especially, resistance against pressure and sliding is credited to the influence of own weight, preloading, and original resistance. The author concludes that the concepts and effects shown in his article are meant only to point up the importance of the mentioned factors.

Ch. Széchy, Hungary

2071. **D. Bonnard and J. Bonjour, Advance estimation of settlement of structures** (in French), Géotechnique Lond. 1, 148–159 (June 1949).

The authors give data about the settlements of four buildings. The settlements were estimated in advance and checked afterwards. The settlement forecasts are based upon oedometer tests on undisturbed samples of the different soil layers.

The samples are taken with a sampler which is forced into the soil at the bottom of a borehole by a built-in ramming device. The sampler further contains a piston that covers the top of the sample, and that can be moved in either direction by suction or by pressure, in order to retain the sample during the sampling operation, or to push the sample into a sample container in which it is sealed properly. In the oedometer tests care is taken to load the sample as the corresponding soil layer under the future building, in order to avoid the necessity of extrapolation.

The stress distribution in the soil is calculated after Froehlich (*Druckverteilung im Baugrund*). Under rectangular foundation slabs the stress is calculated in the verticals (M) in the center of the slab, in the corner (A) and in the middle of both sides (B and C). On every horizon the stress σ_m is calculated as:

$$\sigma_m = {}^1/_{12}\,(8\,\sigma_M + 2\,\sigma_A + \sigma_B + \sigma_C).$$

The probable duration of the settlements is calculated according to Terzaghi-Froehlich (*Setzungen von Tonschichten*).

The correlation between the calculated and the measured settlements is satisfying, which is attributed to the precision with which the sampling operations, the calculation of the stress in the soil before and after the execution of the building, and the oedometer tests were performed. F. C. de Nie, Holland

2072. Friedrich Schiel, An approximate theory of settlement (in German), Abh. Bodenmech. Grundbau, 14–18; Berlin, Erich Schmidt Verlag (1948).

The problem of settlement of infinitely rigid foundations, with a uniform distribution of stresses and with a uniform settlement is treated. Some comparative results for settlement of various size foundations are given, and applicability of this approximate theory for eccentric loading is pointed out. Ed.

2073. C. E. Benito, An elastic treatise of circular footing slab resting on the surface of a semi-indefinite soil and loaded with a rigid massive concentric with the slab, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 7, 25–27 (1948).

Paper deals with deflections of a circular slab supported by an elastic semi-infinite (Boussinesq-Cerruti) body and loaded by an infinitely rigid loading member on a concentric circular area. Paper does not offer any workable suggestions on how to achieve compliance between soil and slab deflections at intermediate points.

Unfortunately, there are many obscure passages in paper, possibly due to inadequate editing of text. Oscar Hoffman, USA

2074. J. M. Griffith and E. H. Woodman, Instrumentation for field measurements of deflections and pressures for airport pavements, Proc. sec. int. Conf. Soil Mech. Found. Engng. 5, 68–73 (1948).

Description is given of operation and installation of electric-recording devices for measuring earth pressure and pavement deflection, as used by the Waterways Experiment Station (WES), Vicksburg, Miss. Utilization of these instruments in several installations is summarized.

The WES earth-pressure cell, as most frequently used, is a disk 12 in. in diam and 1 in. thick, embedded in the earth mass with extreme care to preserve orientation and soil homogeneity. Earth pressure on cell sets up a fluid pressure in an oil film between two plates, which in turn causes bending of a small internal diaphragm. This bending is indicated on a Wheatstone bridge or oscillograph by means of a compensated resistance-wire strain gage cemented to diaphragm. Calibration is by air pressure against rubber diaphragms on both sides of the cell. Present WES cells are considered to be unsatisfactory for the measurement of pressures over a long period of time, but short-time measurements are judged reasonably accurate and reliable. Development of a cell for long-time measurements is now under way at WES.

Of two deflection gages described, one utilizes as indicating mechanism a pair of Selsyn motors; the other indicates by means of resistance-wire strain gages attached to a cantilever beam which deflects due to the movement measured.

C. Martin Duke, USA

2075. Jean Ferrandon and Pierre Theobald, The flow pressure in a two dimensional isotropic medium (in French), Génie civ. 125, 387–389 (1948).

In a previous 1948 paper on equilibrium of permeable soils, authors introduced the notion of "flow pressure" (the tangential force developed parallel to flow lines) and showed its influence on the general equations of equilibrium.

Here the general expression for flow pressure is given and application is made to stability of slopes. An approximate formula for limit slope of a saturated soil is established, and a diagram gives this inclination as a function of characteristic elements of soil. Aurel A. Beleş, Rumania

2076. A. H. Naylor and I. G. Doran, Precise determination of primary consolidation, Proc. sec. int. Conf. Soil Mech. Found. Engng. 1, Sec. 1d, 34–40 (1948).

Paper presents a new method for determining end point of primary consolidation from laboratory time-consolidation curves. Author compares his method with existing methods, criticizes Casagrande's method as being purely conventional, and says Taylor's method is vulnerable in assuming no secondary effect at time of 90 per cent primary consolidation. New method is based on making a plot of time against log of one, minus degree of consolidation from 60 to 80 per cent linear by adjusting initial and final points of primary consolidation. While difference between existing methods and the one proposed is not great, these differences are considered important when measuring amounts of secondary consolidation. Edward S. Barber, USA

2077. P. A. Coenen, Fundamental equations in the theory of limit equilibrium, Proc. sec. int. Conf. Soil Mech. Found. Engng. 7, 15–20 (1948).

A mathematical treatment is presented of condition of shear in a soil mass, based on theory of limit equilibrium. Based on assumptions made, it is shown that possible lines of failure in a homogeneous earth mass can be represented by a series of concentric circles but not necessarily limited to this series. It is demonstrated that a system of logarithmic spirals and the system of homothetic circles proposed by Jaky are not theoretically justifiable as possible lines of failure.

Woodland G. Shockley, USA

2078. B. S. Browzin, Upon the coefficient of stability reserve of anchored bulkheads, Proc. sec. int. Conf. Soil Mech. Found. Engng. 7, 127–133 (1948).

Experiments were made on an anchored sheet-pile bulkhead 0.15 cm thick, 30 cm wide, and up to 36 cm long, partly buried in sand. Critical depths of penetration were observed with and without various surcharges of weights. Calculated moment of active earth pressure at failure is compared with the sum of resisting moments, due to passive earth pressure plus friction on side walls. Zero friction was assumed on bulkhead, and side-wall friction was based on active earth-pressure ratio rather than pressure ratio for a plane of shear. With above assumptions, it is concluded that calculated resistance is only half the actual.

Edward S. Barber, USA

2079. R. Haefeli, Shearing strength and equilibrium of soils. Shearing strength and water content, Proc. sec. int. Conf. Soil Mech. Found. Engng. 3, 38–44 (1948).

Paper summarizes shearing theory as of 1948 and presents new facts and theory regarding shearing resistance of unsaturated fine-grained soils. Theory for saturated soils is presented more fully in paper by Schaerer, Schaad, and Haefeli, "Contribution to the shearing theory," in same publication.

Experiments were conducted with kaolinite, alumina, and quartz fine-grained soils to investigate relation of shearing resistance to water content at various normal pressures on drained and unsaturated samples. Shearing resistance for plastic kaolinite increased to and passed a maximum with increasing water content, after which it fell off rapidly to a minimum as saturation was approached. Theory indicates the process to be irreversible. Shearing resistance for nonplastic alumina was greatest when saturated, least when dry. Quartz gave an intermediate result. A theoretical expression for saturation limit is developed in terms of shearing resistance and consolidated water content.

C. Martin Duke, USA

2080. **N. N. Verigin, On unsteady motion of soil waters in the vicinity of water reservoirs** (in Russian), Doklady Akad. Nauk SSSR 66, 1067–1070 (June 1949).

The nonlinear partial differential equation of unsteady seepage, used in this paper, is based on the assumption that the horizontal component of the velocity of flow is independent of depth. The paper presents an approximate solution of this equation for the flow parallel to XZ-plane, by means of replacing the variable head in one of the terms by a constant, and thus reducing the equation to a linear form of Fourier type. This reduction, however, unlike the usual one, makes the solution agree with the conditions of steady flow. The solution is applied to two specific cases. No numerical results are given.

Alexander Hrennikoff, Canada

2081. **N. N. Verigin, Seepage of water from an irrigation canal** (in Russian), Doklady Akad. Nauk SSSR 66, 589–592 (June 1949).

The paper presents a solution, by means of conformal mapping, of the two-dimensional problem of seepage from an irrigation canal into previous soil of constant permeability, when the water level in the canal coincides with the horizontal ground surface. Seepage in the capillary zone is included. The shape of the cross section of the canal is determined from the resultant equation, and it resembles a semicircle.

The most significant quantities found are the total seepage discharge and the width between the extreme flow lines both at the surface and at a great depth in the ground, where the flow becomes vertical and the velocity becomes equal to the coefficient of permeability.

A numerical example shows that in the usual conditions, when the width of the canal is 0.20 to 0.30 meters, and the capillary head 1 to 2 meters, the capillary forces increase the discharge some 9 to 12 times compared to the condition when they are absent.

Alexander Hrennikoff, Canada

2082. **P. Filchakov, Electromodeling of seepage problems in heterogeneous soils** (in Russian), Doklady Akad. Nauk SSSR 66, 593–596 (June 1949).

The paper outlines the electrical analogy method of determining the seepage flow nets through dams made of materials with two different coefficients of permeability. The model utilizes pieces of cardboard impregnated with salts of different concentrations, and glued together with a lap joint. Some test results are reported.

Alexander Hrennikoff, Canada

2083. **N. N. Verigin, On the question of the calculation of underground water collectors in conditions of a plane movement of soil waters** (in Russian), Doklady Akad. Nauk SSSR 64, 183–186 (Jan. 1949).

Paper presents solution of an involved problem of seepage in vicinity of a gallery, from which water is being pumped out. Gallery is located below top surface of basic water-bearing stratum of great thickness, overlaid by a semipervious stratum, which itself is covered by upper water-bearing layer, with water head in it different from basic layer. On assumption of somewhat questionable boundary conditions, problem is solved by conformal transformation. Solution is left in a complex form. No numerical examples are given.

Alexander Hrennikoff, Canada

2084. **Takeo Mogami, The mechanics of grains** (in Japanese), Rep. Inst. Sci. Technol. Tokyo 3, 168–170 (May–June 1949).

The relation between the pressure p and volume V of a mass of sand under compression is derived theoretically. First, energy consideration gives $-p\Delta V = \Delta U + \Delta Q$, where ΔV is the volume change, ΔU the increment of internal energy due to elastic deformation of sand grains, and ΔQ the energy dissipation due to mutual friction of sand grains. Next, based on the consideration that Q is an additive quantity depending on the configuration of grains, the author assumes, in analogy with entropy, that $Q = K_1 \log W$, where K_1 is a certain constant and W is the probability of the configuration of grains. On the assumption that the sand mass consists of N grains of equal volume, of which N_1 grains occupy a volume v_1, N_2 grains a volume $v_2, \ldots$, W is determined as the maximum probability of the configuration under the condition $v_1 + v_2 + \ldots = V$, $N_1 + N_2 + \ldots = N$. The calculation proceeds just as the calculation of entropy in statistical mechanics. Further, elastic energy is assumed in the form

$$U = {}^1/_2 K_0(V - V_0)^2.$$

Final expression is $p = K_0(V - V_0) + NK_1(V^{-1} - V_0^{-1})$. Here V_0 is the initial volume corresponding to $p = 0$. The relation is found to be in good agreement with the author's experimental results on the compression of sand in a box.

Isao Imai, Japan

2085. **P. Ya. Polubarinova-Kochina, On some unsteady motions of soil waters** (in Russian), Izv. Akad. Nauk SSSR, Ser. tekh. Nauk 1949, pp. 838–846 (June 1949).

The author discusses the difficulties inherent in problems of two-dimensional unsteady seepage, and describes the experimental study of the following two problems: (1) Flow in the vicinity of drains. (2) Flow between a canal and a soil bank, as the level of the water in the former is suddenly changed.

The experiments have been performed on the actual models and the capillary analog, involving the flow of glycerine between two glass plates. The results are compared with the mathematical solutions, both more accurate and simplified. Some mathematical formulas are given.

Alexander Hrennikoff, Canada

Papers in Proc. sec. int. Conf. Soil Mech. Found. Engng. 7 (1948):

2086. **Nabor Carrillo, Influence of artesian wells in the sinking of Mexico City**, pp. 156–159.

2087. **R. R. Proctor, An approximate method for predicting the settlement of foundations and footings**, pp. 154–155.

2088. **M. S. Youssef, A graphical solution of Stokes' law**, pp. 50–52.

2089. **J. L. McRae, Methods and techniques for the preparation of soil test specimens**, pp. 42–44.

2090. **S. J. Johnson, Failure of an excavation slope**, pp. 107–112.

2091. Jacob Feld, Soil resistance to moving pipes and shafts, pp. 149–153.

2092. D. W. Taylor, Field measurements of soil pressures in foundations, in pavements, and on walls and conduits, including a review of work of the fact finding survey and other field investigations of the Corps of Engineers of the U. S. Army, pp. 84–89.

2093. Charles Senour and W. J. Turnbull, A study of foundation failures at a river bank revetment, pp. 117–121.

2094. Gregory P. Tschebotarioff and John David Welch, Lateral earth pressures and friction between soil minerals, pp. 135–138.

2095. William R. Perret, Electrical resistivity exploration as a complement to boring in deep alluvial deposits, pp. 80–84.

2096. James O. Patrick, Stability analysis by application of the elastic theory, pp. 102–107.

2097. T. William Lambe, The measurement of pore water pressures in cohesionless soils, pp. 38–41.

2098. J. J. Kolbuszewski, General investigation of the fundamental factors controlling loose packing of sands, pp. 47–49.

2099. M. Juul Hvorslev, Foundation exploration. A review of methods and requirements, pp. 67–75.

2100. M. Juul Hvorslev, Pocket-size piston samplers and compression test apparatus, pp. 76–79.

2101. Paul Baumann, The significance of prestress in embankment dams, pp. 92–99.

Papers (in German) in Abh. Bodenmech. Grundbau, Berlin, Erich Schmidt Verlag (1948):

2102. Hans Leussink, Recent experiments with the vibration-pressure method, pp. 182–187.

2103. Rudolf Hoffmann, Contribution to the question of static and dynamic bearing capacity of piles, pp. 150–156.

2104. Edgar Schultze, On the question of water pressure on banks, cofferdams and masonry dams, pp. 124–134.

2105. Friedrich H. Siemonsen, The load-bearing forces in building ground and their influence on the stresses in a foundation, pp. 120–123.

2106. Philipp Ebert, Measurement of pressure distribution in building ground with pressure cushion, pp. 114–119.

2107. Heinz Muhs, Practice and result of a large test loading, pp. 97–113.

2108. Hans Lorenz, State of the method of dynamic soil exploration, pp. 81–87.

2109. H. Leussink, Experiments on the deformation of layer soil-bodies in undisturbed bedding at shear loading, pp. 72–80.

2110. Heinz Muhs, A new sampling apparatus for undisturbed soil samples, pp. 69–71.

2111. Peter Siedik, Apparatus for taking undisturbed sand samples, pp. 66–68.

2112. Alfred Ducker, A proposal for the nomenclature of grain sizes, pp. 59–62.

2113. Friedrich Schiel, Earth pressure on lateral wings, p. 46.

Micromeritics

(See also Rev. 2084)

2114. Max Leva, Murray Weintraub, Milton Grummer, and Morris Pollchik, Fluidization of an anthracite coal, Indust. Engng. Chem. 41, 1206–1212 (June 1949).

Fluidization of granular beds by upward flow is a phenomenon of importance in a number of professions. Although paper is concerned primarily with its practical application to industrial catalysis, several aspects of experimental investigation may prove of basic interest. Principal among these are: indication of a difference between air flow and water flow through very fine material, and approximate evaluation of effects of particle shape and porosity. Nonfluidized phase is well annotated; for limit of complete fluidization, reviewer suggests addition of his own paper, "Suspension of sediment in upward flow" [University of Iowa Studies in Engineering, Bulletin 26 (1941)].

Hunter Rouse, USA

2115. G. L. Matheson, W. A. Herbst, and P. H. Holt, Characteristics of fluid-solid systems, Indust. Engng. Chem. 41, 1099–1104 (June 1949).

Authors study the dense phase of gas-solid systems in which solid material was present in finely divided form and was suspended in an upward stream of the gas. Effects of particle size and particle density on maximum bed density were measured and were expressed in an empirical equation. Stormer viscosity of fluidized beds was measured at a single shear rate, and results were shown to be related to flow behavior of the beds.

R. H. Wilhelm, USA

2116. O. H. Hariu and M. C. Molstad, Pressure drop in vertical tubes in transport of solids by gases, Indust. Engng. Chem. 41, 1148–1160 (June 1949).

Paper presents measurements of transport of a stream of solid particles upward in vertical, tubular ducts by means of an air stream. Tube diameters were 0.27 and 0.53 in. in diam, and sand particles were between 28 and 80 mesh sizes. Air velocity was varied between 12 and 40 ft per sec. A systematic analysis was made of component heads, and results were compared experimentally with measured pressure drops. It was found that terminal velocities had not been completely achieved in the measuring duct section.

Richard H. Wilhelm, USA

2117. M. S. Volínskiĭ, On the disintegration of liquid drops in an air stream (in Russian), Doklady Akad. Nauk SSSR 62, 301–304 (1948).

Experimental investigation in which individual drops 2 to 3.9 mm in diameter were dropped into an air jet to investigate factors affecting disintegration of the drops. The parameter $D = \rho V^2 d/\sigma$, where ρ is air density, V jet velocity, d diameter, and σ capillary constant, was found to be significant for drop sizes involved. For $D < 10.7$ no disintegration occurred; for

$10.7 < D < 14$ disintegration was partial, i.e., near lower limit a few drops split in half followed by further splitting. For $D > 14$ the drops split immediately into many drops. Drops of mercury, water, tetrabromoethane, kerosene, ethyl alcohol, and gasoline were used in the experiments. Reynolds numbers for the drops were between 1700 and 8500. A. M. Kuethe, USA

2118. **M. S. Volfnskiĭ, Investigation of drop disintegration in a gas stream** (in Russian), Doklady Akad. Nauk SSSR 68, 237–240 (1949).

This work continues that described in preceding review. Smallest drop size investigated was 0.273 mm, so that this and the previous work covered range of diameters from 0.273 mm to 3.9 mm. A specially constructed drop generator was used which produced drops of a given size with an accuracy of ±0.01 mm. A rough theoretical analysis gives $\rho V^2 d/\sigma = D/[1 - (d_{\min}/d)^{1/2}]^2$ where $10.7 < D < 14$ defines range of partial disintegration as found in previous work, and $d_{\min}$ is the limiting drop diameter for which no disintegration occurs. By use of above values of D and the empirically determined $d_{\min}$, good agreement between theory and experiment was found. A. M. Kuethe, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 1852, 1978)

2119. **R. A. Boutet, The upper atmosphere** (in French), Off. nat. Étud. Rech. aéro. Rep. 38, 121 pp. (1950).

Survey paper that touches lightly upon many subjects. Topics range from cosmic radiation, ionosphere, and northern lights to thermodynamics, geostrophic wind, and even biological aspects. Discussions are uneven and incomplete. There is nothing here that is not treated more soundly and completely elsewhere. Herbert Riehl, USA

2120. **N. P. Sellick, Equatorial circulations,** Quart. J. roy. met. Soc. 76, 89–94 (Jan. 1950).

A cellular flow pattern is presented for which equations of frictionless horizontal nondivergent motion in equatorial regions can be solved formally, and the relationship between pressure field and air trajectories is examined in four cases for which are taken different numerical values for velocity of translation of the pattern and its position relative to the equator. Such particular cases are undoubtedly of great value because no general solution of the equations exists.

Patterns illustrated show that cyclonic circulations predominate and anticyclonic circulations are not obviously associated with any closed isobars. Paper is straightforward, readable, and concise and should be of value to anyone seeking to understand air motion in the lower levels in lower latitudes. The reader may insert his own values of the parameters in the formula to obtain new results, though not without considerable computation, which, however, is straightforward. R. S. Scorer, England

2121. **M. Kiveliovitch, A note on Kibel's method of weather prediction** (in French, with English and Spanish summaries), J. Sci. Météorologie 1, 72–74 (1949).

This note criticizes a method for the numerical integration of the meteorological equations proposed by Kibel. Kibel's equations postulated no vertical motion, geostrophic wind, and adiabatic temperature changes. The present note shows that, if the unrealistic assumption is made of a lapse rate of temperature constant in space, Kibel's equation leads to the paradoxical result that pressure and temperature do not change locally.

Courtesy of Mathematical Reviews H. Panofsky, USA

2122. **R. Berggren, B. Bolin, and C.-G. Rossby, An aerological study of zonal motion, its perturbations and break-down,** Tellus 1, no. 2, 14–37 (May 1949).

It has often been observed that during the winter season the general atmospheric circulation in middle latitudes of the northern hemisphere fluctuates between strong zonal motion and a quasi-stationary meridional pattern of cyclonic and anticyclonic vortexes. In this paper, observational data are presented for a special well-developed case of zonal motion and its final break-down. Observations suggest that the rapidly moving frontal waves with average periods of 33 hr are induced manifestations of relatively short, rapidly moving upper-level waves associated with a jet stream. The velocity of the upper current decreases eastward, this leading to slowing down of the frontal waves, decrease of wave length, increase of amplitudes, and, finally, occlusion over central Europe. This occlusion process does not affect the steering current.

This apparently stable flow finally breaks down through the combined action of a westward moving "blocking wave" and a series of rapidly deepening disturbances originating in the western part of North America. The blocking wave arises from a split in the basic zonal current over the Baltic, its total displacement westward of about 55 deg longitude takes place in eight days. The period of the simultaneous apparently unstable deepening waves is about 70 hr.

Many interesting details and a full discussion in the light of modern theories are included. Horst Merbt, Germany

2123. **W. Schaffer, Changing pressure systems and the ascent of air,** J. Met. 6, 212–215 (June 1949).

The vertical motions accompanying the movement and growth of large-scale atmospheric pressure systems, although usually small compared to the horizontal wind, are of vital significance in forecasting because they play a major role in determining the observed weather.

The present paper derives a simple and useful way of estimating the sign and magnitude of these vertical motions in terms of easily measured or estimated quantities, namely, the speed and rate of development of the system, the rate of turning of the wind with height, the wind speed, and the pressure tendency. This derivation is kinematic, using the gas laws and the hydrostatic equation, and the only restrictive assumption is that vertical accelerations are small. Joanne Starr Malkus, USA

2124. **Lester F. Hubert, High tropospheric westerlies of the equatorial West Pacific Ocean,** J. Met. 6, 216–224 (June 1949).

The paper combines streamline analysis on constant level charts in low latitudes with contour analysis on constant pressure charts in temperate latitudes. The presence of large-scale westerly currents in the high troposphere over the equatorial Western Pacific is illustrated with 40,000-ft charts for the period September 22–27, 1945. These currents are subject to aperiodic fluctuations due to the exchange of air and vorticity across the equator. The relation between the upper currents of both hemispheres and interaction of upper currents and surface weather is illustrated. L. J. Tison, Belgium

2125. **Norman A. Phillips, The work done on the surrounding atmosphere by subsiding cold air masses,** J. Met. 6, 193–199 (June 1949).

The areas of transformation of potential energy in the atmosphere into kinetic have long been supposed to be the cold domes, or surface high-pressure areas. Recently it has been hypothesized that in their subsidence, these cold domes do work on the surrounding air, thus in part at least suggesting a mechanism for the energy transfer.

The present paper represents a considerable advance since it demonstrates theoretically that under the usual conditions this transfer should occur, sets up the equations in a manner suitable for computation, and carries out calculations in a typical case. It is shown that the magnitude of the energy transferred per day is roughly half of that used up against friction by the belt of westerlies during the corresponding period. The author points out, however, that there is no immediate way of determining what fraction of this released energy actually does go into the maintenance of the westerlies, and how much must be consumed on the spot, such as that, for example, needed to raise the center of gravity of the warm air. Joanne Starr Malkus, USA

2126. Archibald Stevenson Thom, Investigation of tidal phenomena in the Clyde estuary, using a scale model, J. Instn. civ. Engrs. 33, 100–125 (Dec. 1949).

Model was constructed to horizontal and vertical scales of 1:5, 280, and 1:115, respectively. Bed of estuary was molded in cement-sand mortar and roughness of this surface was not altered: spring tides in model correspond with spring tides in estuary, but magnification of range of neap tides was insufficient because friction in model was greater at low water than at high water. Tides were generated by a displacer moved by mechanism designed on the principle of Lord Kelvin's tide-predicting machine. Reclamation of shallows in model estuary altered shape of wave but had little effect on range of tide at Glasgow. Observations taken immediately after stopping motion of displacer indicated that a standing oscillation occurs in model with a natural period of oscillation of 29.2 sec, corresponding to a natural period of the estuary of 4 hr. This period corresponds with one third of the period of the tidal wave, resulting in the magnification, due to resonance, of the third harmonic component of tidal wave, and therefore causing a hump on the rising tide in the upper reaches of the estuary. The harmonic analysis of each tidal curve in the estuary gives the mean level of the surface at each point: this mean level at Glasgow is considerably above that at Gourock. The rise is caused by the decrease in specific gravity of the water. L. J. Tison, Belgium

2127. Hsiao-lan Kuo, Dynamic instability of two-dimensional nondivergent flow in a barotropic atmosphere, J. Met. 6, 105–122 (Apr. 1949).

The paper presents an excellent mathematical study, well organized in sections on (a) perturbation equations; (b) analytic properties of the stream function: (c) neutral waves; (d) necessary condition for occurrence of unstable waves; (e) amplified waves; (f) examples with sine-curve velocity profile; (g) dynamic instability of geostrophic flow for three-dimensional perturbations in a baroclinic atmosphere; (h) synoptic examples. A well-written summary reviews the steps of analysis and development presented.

Specifically, in the study of wave motions along a strong jet of westerlies the possibility is considered of having a phase velocity whose value is between the maximum and minimum west-wind velocities, extending it from the very long and slowly moving or retrograding waves into the realm of ordinary waves and cyclone waves. For nondivergent barotropic motion, the condition for the presence of neutral and amplified waves of such phase velocity is the existence of critical points where the absolute vorticity has an extreme value.

In the absence of such a point, all perturbations must be damped; when this condition is satisfied, both amplified and neutral waves can be expected. The wave moving with a velocity equal to the current velocity at the critical point is neutral, while those with a velocity between this value and the minimum wind velocity will be amplified. Amplification is greatest for intermediate velocities. The degree of instability also depends on the sharpness of the velocity profile. When the wave is unstable, the through line will be directed from southeast toward northwest to the south of the point of minimum absolute vorticity and from southwest toward northeast to the north of the point of maximum vorticity.

Treated in an appendix are: (a) the amplified waves in the vicinity of the neutral solution at the critical point, and (b) the amplified waves in the vicinity of the stationary neutral solution. Hans F. Winterkorn, USA

Lubrication; Bearings; Wear

(See also Rev. 1841)

2128. F. T. Barwell and J. A. Webber, The influence of combined thrust and radial loads on the performance of high speed ball bearings, Proc. seventh int. Congr. appl. Mech. 4, 257–267 (1948).

Paper records experimental work which indicates temperature rise in high-speed ball bearings as a function of direction of load with respect to axis of shaft. As a result, it is also possible to indicate permissible load for a given temperature rise as a function of direction of the resultant of load. Paper is important because it gives experimental results which indicate that some additional theoretical work must be done. Friction torque of bearings tested is also indicated, and its variation with load and load direction noted. Erle I. Shobert, II, USA

2129. J. T. Burwell, The calculated performance of dynamically loaded sleeve bearings—II, J. appl. Mech. 16, 358–360 (Dec. 1949).

In a previous paper of same title (Rev 1, **557**) author calculated path of center of a Diesel-engine crankpin based on equations for no side leakage, and used an approximate method in which experimental data for statically loaded 120-deg partial bearings are utilized to account for leakage.

Since then, performance data calculated by Cameron and Wood of a nature more applicable to above bearing have become available, and author recalculates in paper the path of the center of crankpin.

Brief outline of method used is as follows: For an infinitely long full journal bearing with a discontinuous oil film, Cameron and Wood found that the dimensionless load $1/S = P/(R/C)^2\mu N$ is inversely proportional to $1 - \eta$, where P = load, R/C = radial clearance ratio, μ = viscosity, N = revs per unit time, η = eccentricity ratio. Author found that bearings of finite length also follow this relation empirically. Thus, plotting $1/S$ versus $1/(1 - \eta)$ for both infinite and finite bearings, the ratio $(1 - \eta_\infty)/(1 - \eta)$ can be established for each value of load. Values of η_∞ are obtained from the differential analyzer as described in previous paper, and corresponding values η can be computed. Discussing numerical results, author concludes that this bearing is operating hydrodynamically with a reasonable margin of safety. Nicholas Sag, Australia